Name: Answer Key

Econ 101 Extra Credit Homework #4

Introduction to Competition

Marginal Revenue when inverse demand is $P = b - mX$	$MR = b - 2mX$	MR: Marginal Revenue

Background:

For much of the 20th century, if you wanted a diamond, you bought it from De Beers, which controlled up to 90% of the diamond market. They owned all the major mines. To begin with, assume De Beers is a monopoly, the only seller of diamonds. Assume the inverse demand curve for diamonds (globally) is given by

P = 650 – X

(where X is measured in millions of carats). Also assume the marginal cost (MC) and total cost (TC) of diamond production is given by:

MC = 50 TC = 50X

Given these assumptions, you can show, using the equation at the top of the page, that MR = 650-2X. Using the MR = MC method, we find

- Profit Maximizing Output: X = 300 million Carats
- Profit Maximizing Price: P = $350/carat
- Total Profit: ($350 - $50)x300million = $90 billion

1. De Beers eventually lost its monopoly as countries decided to no longer sell diamonds through De Beers. Today it controls about 40% of the Diamond market. Suppose that there are now two diamond producers, De Beers and Alrosa (the Russian distributor of diamonds). Assume they face the same inverse demand, and that Alrosa has the same marginal and total costs as De Beers. Moreover, the two compete by deciding a quantity to supply, and then letting demand determine the final price.
   a. If Alrosa and De Beers each sell 150 million carats (half the monopoly amount), how much profit do they each make?

      $45 billion (half of $90 billion monopoly profits)

b. Assuming Alrosa continues to sell 150 million carats, what is the inverse residual demand curve faced by De Beers?

P = 650 – (150 + D)
P = 500 – D

c. What is the marginal revenue curve implied by this residual demand curve?

MR = 500 – 2D

d. How many carats should De Beers sell, in order to maximize profit?

MR = MC
500 – 2D = 50
D = 225 million carats

e. If Alrosa sells 150 million carats and De Beers produces the amount you found in part (d), what price does a carat sell for?

P = 500 – 225 = $275 or
P = 650 – (225 + 150) = 650 – 375 = $275

f. How much profit does De Beers make?

(275 – 50) x 225 = $50.625 billion (or rounded)

g. How much profit does Alrosa make?

(275 – 50) x 150 = $33.75 billion (or rounded)

h. Can you give an intuitive explanation for why De Beers has its profit rise (compared to part (a)) and Alrosa has its profits fall?
De Beers sells more, which causes prices to fall. This reduces Alrosa's profit. De Beers makes more profit, because the amount it sells is sufficiently high to offset this decline in the price.

2. Alrosa will not sit idly by in this situation. Assume De Beers continues to produce the amount you found in part 1(d).
   a. What is the inverse residual demand curve faced by Alrosa?

   P = 650 – (A + 225) = 425 – A

   b. What is the marginal revenue curve implied by this residual demand curve?

   MR = 425 – 2A

   c. How many carats should Alrosa sell, in order to maximize profit?

      MR = MC
      425 – 2A = 50
      A = 187.5 million carats

   d. How much profit does Alrosa make?

      P = 425 – 187.5 = $237.50 or P = 650 – (225 + 187.50) = 237.50

      (237.50 – 50) x 187.5 = $35.15625 billion (or rounded)

   e. How much profit does De Beers make, if it continues to produce the same amount as you found in part 1(d)?

      (237.50 – 50) x 225 = 42.1875 billion

3. De Beers can now respond to Alrosa. And then Alrosa can respond. Assume Alrosa supplies 200 million carats.

   a. How many carats should De Beers sell in order to maximize profits?
      P = 650 – (200 + D)
      P = 450 –D
      MR = 450 – 2D
      450 – 2D = 50
      D = 200
   b. How much profit does De Beers make?
      P = 650 – 400 = 250 or P = 450 – 200 = 250
      (250 – 50) x 200 = $40 billion

   c. How much profit does Alrosa make, if it supplies 200 million carats?

      $40 billion (same output and price as above)

4. Suppose Alrosa makes the following threat to De Beers: "I will sell 150 million carats worth of diamonds every year, for so long as you do the same. If you sell more than 150 million carats, I will punish you by selling 600 carats per year for 9 years."

Computing PDV: Step One	$\frac{C}{R/100}\left[(1+R/100)^{Y+1}-1\right]=B$	C: Yearly Contribution R: Interest Rate Y: # of Years Invested B: Balance after Y years
Computing PDV: Step Two	$PDV=\frac{B}{(1+R/100)^{Y}}$	PDV: Present discounted value B: Balance computed from the "Savings with Interest and Yearly Contributions" equation R: Interest Rate Y: # of Years

   a. If Alrosa sells 600 carats per year and De Beers sells 0 carats, how much profit do they each make per year?

      P = 650 – 600 = $50
      (50 – 50) x 600 = $0. They each make zero.

b. If De Beers goes along with Alrosa's offer, it will produce 150 million carats per year. If De Beers discounts the future at 3.5%, what is the PDV of De Beers profit over the next 10 years? Use your answer to question 1(a) for the annual profit amount.

$$\frac{45}{0.035}\left[1.035^{11}-1\right]=\$591.389 \text{ billion}$$

$$\frac{591.389}{1.035^{10}}=\$419.247 \text{ billion (or rounded)}$$

c. If De Beers sells more than 150 million carats, it will be punished by Alrosa for 9 subsequent years, earning the answer from part 4(a) each year after the first. For the first year, it can earn the answer from part 1(f). What is the PDV of this strategy over the next 10 years?

$50.625 billion + $0 = $50.625 billion

d. What should De Beers do?

It should cooperate with Alrosa.

5. Let's imagine that a new technology becomes available that lets anyone make diamonds in their garage. The marginal cost for making diamonds with this new wonder-technology is:

   MC = q/12

   Where q is measured in carats (not millions of carats). The inverse demand for diamonds, measured in carats, rather than millions of carats, is:

   P = 650 – q/1,000,000

   Assume there are 1,000,000 small diamond manufacturers, and they have driven De Beers and Alrosa out of business. Each diamond manufacturer sells 600 carats.

a. Consider the 1,000,000th diamond manufacturer. What is the residual demand curve he faces, if the remaining 999,999 diamond makers sell 600 carats each?

$$P = 650 - \frac{x + 999,999 \cdot 600}{10^6} = \frac{650,000,000 - 599,999,400}{10^6} - \frac{x}{10^6} = \frac{50,000,600}{10^6} - \frac{x}{10^6}$$

(or equivalent)

b. What is the MR curve faced by the 1,000,000th diamond manufacturer?

$$MR = \frac{50,000,600}{10^6} - \frac{2x}{10^6}$$ (or equivalent)

c. What is the profit maximizing amount that he should sell?

$$\frac{50,000,600}{10^6} - \frac{2x}{10^6} = \frac{x}{12}$$

$$50,000,600 = \left(2 + \frac{10^6}{12}\right)x = 83,335.333x$$

$$599.993 = x$$

d. What is the price for carats of diamonds if he sells this much?

$$P = \frac{50,000,600 - 599.993}{10^6} = \frac{50,000,000.007}{10^6} = 50.000000007$$

e. How different is your answer to parts (c) and (d) if you assume the MR curve he faces is MR = 50?

$$50 = \frac{x}{12} \rightarrow x = 600$$

$$P = \frac{50,000,600 - 600}{10^6} = 50$$

f. Construct an individual supply curve for a diamond manufacturer, by setting P = MC

P=q/12 → q=12P

g. Construct the aggregate supply curve for diamonds

Q = 12,000,000P