Extra Credit Homework #1 *Answer Key*

Interest Rates

Key Formulas:

Formula Name	Formula	Variable Definitions
Compounding Interest	$P \times (1 + R/100)^Y = B$	P: Principle R: Interest Rate Y: # of Years invested B: Balance after Y years
Income from Interest	$\frac{P \times R}{100} = I$	P: Principle R: Interest Rate I: Income
Present Discounted Value	$PDV = \frac{V}{(1 + R/100)^Y}$	PDV: Present discounted value V: Value in Y years R: Interest Rate Y: # of Years
Savings with Interest and Yearly Contributions	$\frac{C}{R/100} \times \left[(1 + R/100)^{Y+1} - 1 \right] = B$	C: Yearly Contribution R: Interest Rate Y: # of Years Invested B: Balance after Y years

1. You inherit $2,000:
    a. If you put it in 3-month treasury bonds with yearly 3.5% interest, how much will it be worth in 30 years?

    $2000 \cdot (1.035)^{30} = 5613.59$ (or rounded)

    b. If you invest it in the stock market, where it earns 7% interest, how much will it be worth in 30 years?

    $2000 \cdot (1.07)^{30} = 15,224.51$ (or rounded)

    c. A 7% interest rate is twice a 3.5% interest rate. Why is the answer to part (b) *not* twice the answer to part (a)?

    Interest compounds.

2. How much principle do you need, if you want to have an income of:
    a. $100,000 per year, and the interest rate is 10%?

$$\frac{100,000}{0.1} = 1,000,000$$

    b. $30,000 per year, and the interest rate is 3%

$$\frac{30,000}{0.03} = 1,000,000$$

3. The Bill & Melinda Gates foundation has an endowment (principle) of $40.2 billion. If this is invested in 3-month treasury bills with a yearly interest rate of 3.5%, how much can the foundation afford to give away every year without shrinking the principle?

$40.2 \cdot 0.035 = 1.407$ billion (or rounded)

4. The Thames Barrier is a flood wall that can be open and closed to protect the city of London. It was built in 1982 at a cost of 534 million pounds. In 2013, the total value of all London property was 1.35 trillion pounds.
    a. In 1982, what would have been the present discounted value of London property, given an average interest rate of 7%?

$$\frac{1350}{1.07^{2013-1982}} = \frac{1350}{1.07^{31}} = 165.74 \text{ billion (or rounded)}$$

    b. Assume we only care about the value of destroyed property and not human suffering. If the Thames barrier was only used for the first time in 2013 to prevent the city from being totally destroyed, would it still have been a good investment? Why?

        Yes. The cost of investment (534 million pounds) is much less than the present discounted value of London property.

5. New York city has recently been hit hard by hurricanes. Hurricane Sandy caused an estimated $19 billion in damages, but by 2055 the city estimates a storm could cause $90

billion worth of damage, since the economic value of the city is projected to increase. The city is considering investing in measures to protect the city:

a. What is the present discounted value of $90 billion in 2055 today, if we use an interest rate of 7%?

$$\frac{90}{1.07^{2055-2014}} = \frac{90}{1.07^{41}} = 5.617 \text{ billion (or rounded)}$$

b. What is the present discounted value of $90 billion in 2055 today, if we use a discount rate of 3.5%?

$$\frac{90}{1.035^{2055-2014}} = \frac{90}{1.035^{41}} = 21.963 \text{ billion (or rounded)}$$

c. If the interest rate goes down, the present discounted value goes up! Why? Try to explain with intuition.

The present discounted value is the amount we need to invest today to have $90 billion in 41 years. If the interest rate of investment is lower, we need more today.

6. How much do you need to contribute to an investment account each year to become a millionaire at age 65 if the interest rate is 7% and you start saving when you are:

a. 15:

$$\frac{c}{0.07}\left[1.07^{51}-1\right] = 1,000,000 \rightarrow c = \frac{0.07 \cdot 1,000,000}{1.07^{51}-1} \rightarrow c = 2,293.65 \text{ or rounded}$$

b. 25:

$$c = \frac{0.07 \cdot 1,000,000}{1.07^{41}-1} = 4,659.62 \text{ or rounded}$$

c. 35:

$$c = \frac{0.07 \cdot 1,000,000}{1.07^{31}-1} = 9,796.91 \text{ or rounded}$$

d. 45:

$$c = \frac{0.07 \cdot 1,000,000}{1.07^{21}-1} = 22,289.00 \text{ or rounded}$$

e. 55:

$$c = \frac{0.07 \cdot 1,000,000}{1.07^{11}-1} = 63,356.90 \text{ or rounded}$$

7. There are no guarantees in life. Suppose you want to be a millionaire at age 65, and you expect the interest rate to be 7%. Much to your disappointment, suppose the interest rate on stocks only averages 4%. Using your answers to question 6 as the yearly contribution, how much will have at age 65 if the interest rate is only 4% and you start saving at age:

   Note – if they use the correct formula on problem 7, don't count problem wrong if they got the wrong answer for question 6, and then used this incorrect answer in question 7

   a. 15:

   $$\frac{2,296.65}{0.04}\left[1.04^{51}-1\right]=366,944.42 \text{ or rounded}$$

   b. 25:

   $$\frac{4,659.62}{0.04}\left[1.04^{41}-1\right]=465,153.73 \text{ or rounded}$$

   c. 35:

   $$\frac{9,796.91}{0.04}\left[1.04^{31}-1\right]=581,234.36 \text{ or rounded}$$

   d. 45:

   $$\frac{22,289.00}{0.04}\left[1.04^{21}-1\right]=712,561.54 \text{ or rounded}$$

   e. 55:

   $$\frac{63,356.90}{0.04}\left[1.04^{11}-1\right]=854,453.42 \text{ or rounded}$$