Mandates and the Incentive for Environmental Innovation

Matthew S. Clancy and GianCarlo Moschini

Abstract

Mandates, which establish minimum use quotas for certain goods, are becoming increasingly popular policy tools to promote renewable energy use. In addition to mitigating the pollution externality of conventional energy, clean energy mandates have the goal of promoting research & development (R&D) investments in renewable energy technology. But how well do mandates perform as innovation incentives? To address this question, we develop a partial equilibrium model to examine the R&D incentives induced by a mandate, and compare this policy to two benchmark situations: *laissez faire* and a carbon tax. Innovation is stochastic and the model permits an endogenous number of multiple innovators. We present both analytical results and conclusions based on numerical simulations. We find that the optimal mandate is larger than it would be without the prospect of innovation, that neglecting the outlook for innovation significantly reduces welfare, and that the optimal mandate is more sensitive to assumptions about the innovation process than an optimal carbon tax. Furthermore, we find mandates create relatively strong incentives for R&D investment in low-quality innovations, but relatively weak incentives to invest in high-quality innovations. We also rank policies by expected welfare. An optimal carbon tax has higher expected welfare than an optimal mandate, and both have higher expected welfare than *laissez faire*. Moreover, in our endogenous innovation setting a stronger result obtains: a simple carbon tax equal to the damage from pollution (unadjusted for the prospect of innovation) has higher expected welfare than an optimal mandate.

Key Words: Carbon tax, Innovation, Licensing, Mandates, R&D incentive, Renewable energy, Second best, Welfare.

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Mandates have emerged as a key policy tool to promote the use of biofuels and other renewable energy. Such policies set a target for renewable energy production, and it falls upon the producers and suppliers of energy to meet this quota. In the United States, the use of mandates is one of the distinctive features of the 2007 Energy Independence and Security Act (EISA), which set out requirements for overall biofuel use as transportation fuel in the United States to grow to 36 billion gallons by 2022 (Stock 2015).[1] These mandates have been effective at spurring the growth of the corn-based ethanol industry, which steadily accumulated the capacity required to produce the mandated targets in a timely fashion (Moschini, Cui and Lapan 2012). But in order to meet the ambitious targets set out by EISA, a major role is envisioned for advanced biofuels such as cellulosic ethanol: 21 of the 36 billion gallons of biofuels mandated by 2022 are supposed to come from advanced biofuels. So far, however, the US production capacity for cellulosic ethanol has severely lagged the originally intended levels. A crucial element in this setting is that corn-based ethanol is produced with a mature technology, and the EISA mandate was essentially meant to force adoption of this technology. Cellulosic ethanol, by contrast, required technological breakthroughs to make it scalable and commercially viable at the time EISA established the mandates schedule. For such advanced biofuels, therefore, mandates were really supposed to spur invention (rather than merely adoption) of innovations. Yet innovation in advanced biofuels appears to be stalling (Albers, Berklund and Graff 2016).

The effectiveness of mandates to promote innovation has been largely ignored by the considerable literature on the economic impacts of biofuel policies.[2] Is the purpose of promoting invention and development of new technologies a reasonable aspiration for a policy tool such as mandates? A careful reading of real-world policy experiences in the related context of technology-forcing policies suggests grounds for skepticism: "*Technology-forcing performance standards have had a mixed record in inducing innovation. Regulators can find it difficult to obtain information about the status of technologies that is accurate enough to allow them to set standards that both can be achieved and will induce real innovation. Such standards may be effective when the path to a technological solution is reasonably clear, but are less likely to be effective in stimulating cost-effective and broad-based breakthrough technologies*" (Arrow et al. 2009). As for theoretical models comparing environmental policy tools in terms of their innovation-inducing potential, they have emphasized the dichotomy of prices versus quantity tools and have privileged the comparison of carbon taxes with (tradable) pollution permits.[3] It is unclear how existing results may apply to mandates, which differ from pollution permits by establishing minimum levels of production with a "clean" technology, rather than maximum levels of production with a "dirty" technology. This distinction matters when the price of energy changes (as it will when innovation changes production costs) because the margin on which energy supply can adjust varies across the policies.

The purpose of this paper is to directly investigate the effectiveness of mandates as a policy tool to promote environmental innovation in the context of an explicit model of private R&D investments. The stochastic innovation model that we develop is meant to capture some essential long-run features of the innovation process and envisions three distinct stages: the choice of policy instrument and its level; the forward-looking decision of innovators to invest in R&D, given the policy context and their information about technological opportunity; and, *ex post* licensing of successful innovations to adopters, followed by production and consumption decisions. Specifically, we consider a market with clean and dirty energy sources that are close substitutes, e.g., renewable energy and fossil fuels. The dirty energy imposes a negative externality on society. The clean energy has no such externality, and the cost of producing it can be lowered through R&D. Following Parry (1995), Laffont and Tirole (1996) and Denicolo (1999), we view the R&D sector as separate from the production sector adopting the new technology. Policies such as mandates can affect R&D because they influence the profit opportunity that motivates innovators. In this setting, no environmental policy measure can lead to a first best outcome by itself. The effectiveness of mandates at spurring environmental innovation, therefore, is best understood as compared to a well-defined alternative. Hence, in this paper we compare the innovation effects of a mandate with that of a carbon tax (the prototypical formalization of a price-based environmental policy).[4]

Perhaps surprisingly, the process of environmental innovation is often modeled deterministically.[5] Also, the simplifying condition that innovation is undertaken by a single agent is often maintained. In our model we relax both of these conditions. Because the crux of the matter is invention of new technologies, rather than adoption/diffusion of existing technologies, we develop an explicit stochastic framework by positing that a firm that invests in R&D gets an independent random draw of a cost-reducing technology for the production of renewable energy. Furthermore, having introduced the problem of a single innovator, the focus of the paper is squarely on the case of multiple innovators. For this purpose, we implement a novel free-entry representation of an innovation contest, following Spulber (2013), where the number of innovators is endogenously determined. In our framework, multiple innovators can raise welfare through two channels: an increase in the number of innovating firms increases the expected quality of the best innovation that will be discovered, and, the *ex post* royalty rate for the best innovation is reduced by the presence of competitors. This formulation also effectively captures the welfare spillover effect of innovations and the associated appropriability problem that is one of the roots of R&D under-provision.[6] Our model also maintains a plausible presumption about the innovation process: by the time they choose R&D investments, firms have better information than policy makers did when they set the policy.

Two additional features of our modeling framework deserve a brief discussion. First, we assume that the marginal environmental damage of the externality is constant. This commonly invoked condition, together with the assumption that the conditional distribution of firms' innovation outcomes is uniform, simplifies the analysis and permits the derivation of explicit results. Besides its analytical attractiveness, this assumption might be appropriate for the case of renewable energy. For example, advanced biofuels can only address a small portion of the overall energy needs of the economy, and innovations in this area are likely to have a limited impact on the overall level of carbon emission. Furthermore, the energy sector's emissions are small relative to the *cumulative stock* of emissions, which is what ultimately drives climate change. Hence, a linear damage function is arguably appropriate in our context. Second, in studying the R&D incentive of mandates we assume that policymakers commit to the level of the chosen instruments. That is, in this paper we do not address the well-known time consistency issue: once new less-polluting technologies are developed, policy makers might want to change environmental rules, and this *ex post* policy adjustment alters the innovator's *ex ante* incentives (Laffont and Tirole 1996, Denicolo 1999, Kennedy and Laplante 1999). Although what assumption about commitment is most appropriate may depend on the real-world policy context of interest, the issue of time consistency is clearly germane. This paper provides a first look at the innovation role of mandate policies, and the study of commitment issues is left for future work.

Our results show that a mandate is relatively good at incentivizing incremental innovation but a poor spur to breakthrough innovation, as compared with a carbon tax. A carbon taxes is more likely to realize either a very good innovation or none at all, whereas mandates induce a comparatively low dispersion of realized technologies (at least some form of innovation is likely to be realized). A mandate can improve upon *laissez faire*, but the prospect of innovation is essential for the desirability of mandates and, unlike a carbon tax, the mandate's level must be carefully tuned to incorporate its expected effects on innovation. In our numerical simulations, carbon taxes consistently achieve higher expected welfare than mandates. Indeed, for the general case of competitive innovation, even the naïve carbon tax also has higher expected welfare than an optimal mandate.

The Model

We view innovation as a purposeful economic activity undertaken by R&D firms seeking to profit from licensing the implementation of their successful ideas. Specifically, we focus on the invention of a new technology to produce cleaner energy. The model envisions two forms of energy: conventional (dirty) energy, denoted Q_1, and renewable (cleaner) energy (e.g., advanced biofuels), denoted Q_2. Innovation reduces the cost of producing clean energy. Consumers are assumed to have quasilinear preferences for a *numeraire* good and energy Q, with the aggregate inverse demand for energy given by $P(Q)$, where $P'(Q) < 0$. The two sources of energy are perfect substitutes from the consumer's perspective, and thus we can represent total energy used as $Q = Q_1 + Q_2$.[7] Given the premise that renewable energy is less polluting than conventional energy, without much loss of generality it is assumed to have zero emissions. Total damage from emissions therefore can be represented as $X = xQ_1$, where x is the (constant) marginal environmental damage rate.

Innovation contexts are inherently dynamic. To capture the salient features of the problem at hand, and yet retain the tractability of a static framework, we develop an "ideas" approach to modeling innovation (Scotchmer 2004, Spulber 2013). Each potential innovator has an idea for a distinct research project that costs k to implement and yields a draw of θ from the conditional distribution function $F(\theta|\omega)$. The parameter θ measures the quality of the cost-reducing innovation, whereas ω characterizes technological opportunity, i.e., the state of scientific and engineering knowledge that can be applied towards the problem of producing renewable energy. This is best thought of as knowledge exogenously developed in relevant fields (e.g., biology, chemistry, material sciences, computer science, etc.) and therefore not responsive to targeted policies such as those considered in this paper.

The structure of the model is represented in Figure 1, where the timeline of decisions (listed at the bottom) is illustrated together with the timeline of information revelation (listed at the top). Innovators first receive a draw of ω from a cumulative probability distribution $G(\omega)$ with domain $[0, \bar{\omega}]$. Given ω, the researcher chooses whether or not to pay k to obtain a draw of θ from the conditional distribution function $F(\theta|\omega)$. Whereas the distribution function $G(\omega)$ is unrestricted, apart from the standard monotonicity and continuity properties, the analytical results that we present rely on postulating that $F(\theta|\omega)$ is a uniform distribution. The density function of this distribution is:

$$(1) \qquad f(\theta|\omega) = \begin{cases} 1/\omega & \text{if } \theta \in [0, \omega] \\ 0 & \text{otherwise} \end{cases}$$

Note that both the expected value and the upper bound of the innovation draw θ are increasing in the technological opportunity parameter. But because even the most promising innovation can fail, the lower bound on innovation quality is always zero.

Whereas innovators are assumed to observe the signal of technological opportunity prior to making the R&D investment, we presume that the policy setting is determined in advance of the realization of this signal. To evaluate and compare policies, therefore, we will take the *ex ante* perspective of policy makers who know the distributions $G(\omega)$ and $F(\theta|\omega)$ but do not know the actual information possessed by innovators. In this setting, we evaluate the effectiveness of mandates as a policy tool to both ameliorate the externality and promote innovation. For a meaningful benchmark, we compare mandates with a carbon tax, and also to the *laissez faire* (no policies) situation. Although each firm undertakes at most one research project, in our competitive innovation framework the aggregate supply of R&D projects responds to changing incentives through the endogenous number of firms who undertake innovation projects.[8]

A distinctive feature of the innovation context that we wish to model is that the renewable source of energy is unlikely to be able to completely supplant the conventional source, and, relative to the latter, it is expected to be at a production disadvantage. Indeed, the issue of scalability is a critical limitation of many carbon-neutral new technologies and renewable energy alternatives, including biofuels, solar and wind (Galiana and Green, 2009). To capture this asymmetry, we assume that the production of the older product displays constant returns to scale at the industry level, whereas renewable energy is produced under decreasing returns to scale at the industry level. Furthermore, whereas the analysis that we present does not restrict the shape of the inverse demand function $P(Q)$, to obtain clear results (especially for the competitive innovation case) we restrict attention to linear industry marginal cost schedules. If $C_1(Q_1)$ and $C_2(Q_2, \theta)$ denote the industry cost functions for the two products, conventional energy is assumed to be produced by a perfectly competitive industry with constant marginal cost, whereas the new clean technology displays an upward-sloping marginal cost function. Specifically:

(2) $$\frac{\partial C_1(Q_1)}{\partial Q_1} = c_1$$

(3) $$\frac{\partial C_2(Q_2,\theta)}{\partial Q_2} = c_2 - \theta + Q_2$$

where c_1 and c_2 are fixed parameters, with $c_2 > c_1$, and θ captures the impact of innovation.[9]

These marginal costs are illustrated in Figure 2. Initially, $\theta = 0$, but, as exemplified in (3), innovation lowers the marginal cost of producing renewable energy. Innovation is understood as producing know-how, and this knowledge is patentable. Innovators produce a blueprint for a new technology, and can license these blueprints to the competitive production sector that produces renewable energy. Licensing is presumed to take the forms of a fixed royalty rate r per unit of Q_2.[10]

Mandates

A mandate policy specifies a minimum amount of renewable energy to be used as part of the production/consumption portfolio: distributors must ensure that $Q_2 \geq \bar{Q}$, where $\bar{Q}$ is the mandated minimum quantity of total renewable energy. The implementation of this mandate postulates the existence of a competitive blending sector that combines energy from two sources: conventional energy, priced at its constant marginal cost c_1, and renewable energy, priced at its (increasing) marginal cost $\partial C_2 / \partial Q_2 = c_2 - \theta + r + Q_2$. The specifics of how the mandate is enforced are not important in our competitive context.[11] What matters is that the extent of the mandate affects the price of blended fuel. The zero profit condition for the competitive blending sector ensures that, for a given mandate $\bar{Q}$ of renewable energy and corresponding quantity $(Q - \bar{Q})$ of conventional energy, consumers are charged a blend price $\tilde{P}(Q)$ that is the weighted average of the energy input costs (de Gorter and Just 2009, Lapan and Moschini 2012):

(4) $$\tilde{P}(Q) \equiv c_1 \frac{Q - \bar{Q}}{Q} + \left(c_2 - \theta + r + \bar{Q}\right)\frac{\bar{Q}}{Q}$$

The issue of feasibility of the mandate should be noted at this juncture. Feasibility is relevant because how much consumers are willing to buy at the blend price is still governed by the (inverse) demand function $P(Q)$. Because consumers (and competitive suppliers) cannot be coerced, not every arbitrary mandate $\bar{Q}$ is feasible. Therefore, we assume the following condition.

Condition 1. The mandate is feasible in that there exists an equilibrium total quantity that solves $\tilde{P}(Q^*) = P(Q^*)$ and satisfies $Q^* \geq \bar{Q}$.

Figure 3 illustrates the case of a feasible mandate ($\bar{Q}'$) and that of an unfeasible mandate ($\bar{Q}''$). For a fixed $\bar{Q}$, and given that $c_2 + \bar{Q} > c_1$, the blend price $\tilde{P}(Q)$ is decreasing in Q and asymptotically approaches c_1 from above as Q increases. Depending on the shape of the inverse demand function $P(Q)$, there may be multiple solutions to $\tilde{P}(Q^*) = P(Q^*)$ (in which case one may appeal to stability conditions to select the relevant equilibrium) or, for unfeasible mandates, none at all.

The formulation in (4) presumes that the mandate is binding, typically the policy-relevant case of interest. The following condition ensure this is the case (we relax this assumption in our numerical analysis section).

Condition 2. The mandate is large enough to always bind, i.e., $\bar{Q} \geq \bar{\omega} - (c_2 - c_1)$.

Note that this sufficient (but not necessary) condition simply requires that the best possible new technology is insufficient to exceed the mandate at a price competitive with fossil fuels.

Laissez faire and the carbon tax

Without a mandate, we must explicitly consider how the quantity of renewable energy supplied varies with the realized innovation θ. For both the *laissez faire* situation (absence of government policy) and the case of a carbon tax, the residual inverse demand curve facing producers of renewable energy can be written as:

$$(5) \qquad P_2(Q_2) = \begin{cases} c_1 + t & \text{if } Q_2 \leq P^{-1}(c_1 + t) \\ P(Q) & \text{otherwise} \end{cases}$$

where t denotes the carbon tax (per unit of dirty energy). For the *laissez faire*, $t = 0$. In such a case, if clean energy is priced below the cost of dirty energy (c_1), then it captures the entire market; if it is priced above the cost of dirty energy, demand for clean energy falls to zero; and, any quantity

$Q_2 \in \left[0, P^{-1}(c_1)\right]$ can be sold when clean energy is priced at the cost of dirty energy.

As noted earlier, the realistic scenario is that the new renewable energy source does not completely replace the pre-existing conventional source. That is, the innovation is "nondrastic" in Arrow's (1962) terminology. By condition 2, it is impossible to produce more than the mandate at a cost competitive with fossil fuels. Because any feasible mandate must satisfy $\bar{Q} < P^{-1}(c_1)$, this also implies all innovations are non-drastic.

First best allocations

Before considering the effects of innovation, it is useful to note an important asymmetry between the two policy tools when in fact innovation is not possible.

> **Remark 1.** A mandate $\bar{Q} > 0$ is incapable of achieving the first best allocation in the absence of innovation.

This result is well known (Holland, Hughes, and Knittel 2009). When innovation is not possible, the first best solution calls for a total energy consumption that satisfies $P(Q_1 + Q_2) = c_1 + x$, and for a level of renewable energy $Q_2 = \max\left\{0, \tilde{Q}_2\right\}$, where $\tilde{Q}_2$ solves $c_2 - \theta + \tilde{Q}_2 = c_1 + x$. These allocations are clearly achieved by the Pigouvian tax $t = x$, but not by any mandate. For example, whenever $c_2 \geq c_1 + x$ the optimal allocation in the absence of innovation requires $P(Q_1) = c_1 + x$ and $Q_2 = 0$. This mix of energy is impossible to achieve with an instrument that mandates $Q_2 \geq \bar{Q}$.

Remark 1 highlights the fact that the prospect of innovation is essential for the (possible) desirability of a mandate policy, which reinforces the main motivation for the analysis of this paper. When we do allow for the possibility of innovation, neither policy instrument alone can achieve the first best allocation—not surprisingly, given that our model combines a pollution externality with innovation externalities. The analysis that follows, therefore, largely pertains to second best outcomes (although first best allocations are used as a benchmark in the numerical analysis section).

R&D with a Single Innovator

To understand how environmental policy tools affect private R&D decisions, it helps to first consider the case when there is only one firm capable of innovating (this assumption is relaxed later). We consider mandates, *laissez faire* and the carbon tax in turn.

Innovation under mandates

To characterize the innovator's decision problem, consider first the licensing stage for an arbitrary innovation of quality θ. The innovator essentially acts as a monopolist with a competitive fringe, and sets the per-unit royalty r to maximize profits conditional on the adoption constraint by the competitive producers of renewable energy. Thus, the innovator's optimal royalty maximizes $r\bar{Q}$, such that $c_2 - \theta + \bar{Q} + r \leq c_2 + \bar{Q}$. This constraint represents the option that clean producers have to meet the mandate by using the pre-innovation technology (for which $\theta = 0$). With a binding mandate, the profit-maximizing license is $r^* = \theta$. The maximum licensing profit attainable by an innovator with technology θ, under a binding mandate, is therefore $\pi_m = \theta\bar{Q}$, and the expected licensing profit of the innovator with technological opportunity ω, denotes $\pi_m(\omega)$, is:

(6) $$\pi_m(\omega) = \omega\bar{Q}/2$$

The lower bound of technological opportunity for which innovation occurs under a mandate, denoted $\hat{\omega}_m$, solves $\pi_m(\hat{\omega}_m) = k$, and therefore $\hat{\omega}_m = 2k/\bar{Q}$. This threshold is increasing in the cost of R&D and decreasing in the mandate. Under a mandate policy, therefore, R&D occurs with probability $1 - G(\hat{\omega}_m)$.

Turning to welfare, once a mandate is imposed, the price and quantity of energy produced are not changed by (nondrastic) innovation, and therefore there is no change in consumer surplus or in the damage from the externality. The producer surplus of clean firms is also unaffected by innovation, because the innovator fully appropriates the reduction in cost brought about by the innovation. Accordingly, the change in welfare due to innovation is purely derived from licensing profits less R&D costs, so that expected welfare under a mandate is given by:

(7) $$E[W] = S_0^m + \Pi_0^m - X_0^m + \int_{\hat{\omega}_m}^{\bar{\omega}} \left(\frac{\omega\bar{Q}}{2} - k \right) dG(\omega)$$

where S_0^m, Π_0^m, and X_0^m denote the pre-innovation levels of consumer surplus, producer surplus of renewable energy firms, and environmental damages, respectively, that occur under the given mandate policy.

Given equation (7), the level of the mandate that maximizes expected welfare is affected by the prospect of innovation. In fact, we find the following.

> **RESULT 1.** The level (stringency) of the mandate that maximizes welfare is increased when the regulator takes into account its impact on innovation.

To see why this is the case, consider the case where innovation is not possible. Still, because of the unpriced externality, use of some renewable energy is desirable. In this case, the optimal static mandate (i.e., ignoring the prospect of innovation), denoted $\bar{Q}_0$, is such that:

$$(8) \qquad \underset{(-)}{\frac{\partial S_0^m}{\partial \bar{Q}}} + \underset{(+)}{\frac{\partial \Pi_0^m}{\partial \bar{Q}}} - \underset{(-)}{\frac{\partial X_0^m}{\partial \bar{Q}}} = 0$$

where the sign of the derivative is given below each term. This optimality condition pins down the optimal mandate in the absence of innovation (assuming the usual concavity conditions are satisfied). However, accounting for the fact that innovation is possible, when setting the mandate, changes the first order condition. The optimal mandate, denoted $\bar{Q}_0^I$, now solves:

$$(9) \qquad Z(\bar{Q}_0^I) \equiv \underset{(-)}{\frac{\partial S_0^m}{\partial \bar{Q}}} + \underset{(+)}{\frac{\partial \Pi_0^m}{\partial \bar{Q}}} - \underset{(-)}{\frac{\partial X_0^m}{\partial \bar{Q}}} + \underset{(+)}{\int_{\hat{\omega}_m}^{\bar{\omega}} \frac{\omega}{2} dG(\omega)} = 0$$

Note that the indirect impact that arises because the policy change affects $\hat{\omega}_m$ vanishes (a consequence of the envelope theorem). It is apparent that, when evaluated at $\bar{Q}_0$, $Z(\bar{Q}_0) > 0$. This implies that welfare, when evaluated at $\bar{Q}_0$, is increasing in $\bar{Q}$, so that the mandate should be increased relative to the optimal mandate without innovation. The intuition for Result 1 is that innovation increases welfare, and a larger mandate increases the incentive to innovate.

Innovation under laissez faire

To characterize the innovator's decision problem under *laissez-faire*, again consider the licensing stage for an arbitrary innovation of quality θ. The innovator's optimal royalty maximizes rQ_2, where the demand from the competitive adopting clean energy sector, for $Q_2 > 0$, satisfies $c_2 - \theta + Q_2 + r = c_1$. When $c_2 - \theta \geq c_1$ there is no strictly positive license fee that can result in any adoption. In such a case, the innovation is insufficient to be cost-competitive with the dirty technology. Thus, licensing only occurs if the innovative step is sufficiently large. More specifically, $\hat{\theta} \equiv c_2 - c_1$ defines the minimum innovative step beyond which the innovation becomes profitable (see Figure 2). Substituting in $\hat{\theta}$ for the clean energy producer's production constraint, licensing revenues can be written as $r\left(\theta - \hat{\theta} - r\right)$ when $\theta \geq \hat{\theta}$. The optimal royalty is $r^* = (\theta - \hat{\theta})/2$, and at this price the quantity licensed is $Q_2 = (\theta - \hat{\theta})/2$. The maximum profit an innovator with technology θ can obtain, when $\theta \geq \hat{\theta}$, is $\pi = (\theta - \hat{\theta})^2/4$ (and, of course, $\pi = 0$ when $\theta < \hat{\theta}$).

A researcher with technological opportunity $\omega \leq \hat{\theta}$ expects zero profit (no possible innovation is viable). For $\omega > \hat{\theta}$ the innovation can still yield zero profit whenever $\theta < \hat{\theta}$, which happens with probability $\hat{\theta}/\omega$, and thus the researcher expects to make positive profit with probability $1 - \hat{\theta}/\omega$. Expected licensing profit conditional on ω, denoted $\pi\left(\omega\right)$, can therefore be written as:

$$(10) \qquad \pi\left(\omega\right) = \left(1 - \frac{\hat{\theta}}{\omega}\right)\left[\frac{1}{4(\omega - \hat{\theta})}\int_{\hat{\theta}}^{\omega}(\theta - \hat{\theta})^2 d\theta\right] = \frac{(\omega - \hat{\theta})^3}{12\omega}$$

A risk neutral innovator will choose to conduct research if this expected licensing profit exceeds the costs of R&D, i.e., when $\pi(\omega) \geq k$. This implies the existence of a threshold $\hat{\omega}$, which satisfies $\pi(\hat{\omega}) = k$, such that innovation is undertaken if and only if $\omega > \hat{\omega} \geq \hat{\theta}$.

To understand how innovation affects welfare we note that, given the presumption that innovation is nondrastic, renewable energy is always priced at c_1. This means that the total quantity of energy Q, and consumer surplus, are not affected by innovation. Instead, innovation affects the share of energy produced by renewable sources, and reduces the *status quo ante* damage from externalities by xQ_2. Accounting for the minimum innovation step, and proceeding analogously to (10), expected clean energy is $E\left[Q_2\right] = (\omega - \hat{\theta})^2/4\omega$. License profits are given in equation (10). Clean producer profits can be shown to be $(\omega - \hat{\theta})^3/24\omega$ in expectation. All told, therefore, expected welfare in the absence of government intervention is

$$(11) \qquad E[W] = S_0 - X_0 + \int_{\hat{\omega}}^{\overline{\omega}} \left\{ \left[\frac{(\omega-\hat{\theta})^3}{12\omega} + \frac{(\omega-\hat{\theta})^3}{24\omega} + x\frac{(\omega-\hat{\theta})^2}{4\omega} - k \right] \right\} dG(\omega)$$

where S_0 and X_0 denote the pre-innovation levels of consumer surplus and externality damage, respectively, and the integral in (11) is the expected contribution of innovation to welfare.

Innovation under a carbon tax

In the *laissez-faire* welfare is suboptimal because, *inter alia*, the uncompensated negative externality means there is excess production of dirty fuel. The canonical solution to an externality of this type is a Pigouvian tax on the dirty fuel, e.g., a carbon tax. Because use of fossil fuels incurs a social cost x per unit, if one ignores the prospect of innovation the tax should be set at $t = x$. We will use this "naïve" carbon tax as the benchmark in our analytical results, and consider the optimal carbon tax (which also accounts for the prospect of innovation) in the numerical section. Performance under a naïve tax is also of interest when the market under question is small relative to all sources of carbon. With a unit tax t on fossil fuel, clean producers face the inverse residual demand curve given in (5). As illustrated in the previous section, some innovations may be of insufficient size to be competitive, so that the characterization of the impact of innovation needs to always account for the probability that an innovation of sufficient size actually materializes. To simplify the exposition, and without much loss of generality, it is convenient to maintain the following.

Condition 3. The pre-innovation renewable energy technology satisfies $c_2 = c_1 + x$.

This parametric case restricts attention to the situation where renewable energy is on the brink of being competitive, provided the externality posed by the dirty technology is appropriately taxed. Condition 2 guarantees that the optimal supply of renewable energy is positive for any $\theta > 0$ (the parameter $\hat{\theta}$ corresponding to the minimum inventive step can be dropped from the analysis).

Given Condition 3, the optimal license fee and equilibrium quantity of renewable energy satisfy $r^* = Q_2 = \theta/2$. The maximum licensing profit for an innovator possessing an innovation of quality θ, given the existence of the carbon tax t, is: $\pi_t = \theta^2/4$. Hence, the innovator's expected profit conditional on technological opportunity, denoted $\pi_t(\omega)$, is given by:

(12) $\pi_t(\omega) = \omega^2 / 12$

Given the existence of a tax t, the threshold $\hat{\omega}_t$ for R&D to be conducted satisfies $\pi_t(\hat{\omega}_t) = k$, and thus $\hat{\omega}_t = \sqrt{12k}$. It is readily verified that this threshold is lower than under *laissez-faire*, i.e., $\hat{\omega}_t < \hat{\omega}$.

Similarly to the *laissez-faire* situation, with a carbon tax a non-drastic innovation does not affect the total quantity of energy nor consumer surplus. Innovation now improves welfare, through its effect on the cost of producing clean fuel, in two ways: license profit to the innovator, and producer surplus to clean energy producers (recall that the innovator behaves as a monopolist who cannot price discriminate). The former was derived in (12). The producer surplus of clean producers can be shown to be $\theta^2/8$, or $\omega^2/24$ in expectation. Combining all elements, expected welfare with innovation, given the carbon tax $t = x$, is:

(13) $$E[W] = S_0^* + \int_{\hat{\omega}_t}^{\overline{\omega}} \left[\frac{\omega^2}{12} + \frac{\omega^2}{24} - k \right] dG(\omega)$$

were S_0^* denotes the pre-innovation consumer surplus under the naïve carbon tax. When compared with (11) we note that the term related to the environmental externality is absent. But welfare is still suboptimal because innovation is underprovided from a social point of view (the appropriability problem is only partially solved by patents, as discussed in Clancy and Moschini 2013).

Mandate vs. carbon tax: an initial comparison

The comparison that is of most interest is between the "optimal" mandate and the "optimal" carbon tax. We cannot characterize this comparison analytically, but we will pursue it in the numerical analysis section. It is instructive, however, to highlight some of the tradeoffs between optimal mandates and naïve carbon taxes. Focus on the naïve carbon tax benchmark is of interest because, as we show in the numerical results presented later, this tax level is actually close to the optimal carbon tax. More generally, the naïve carbon tax may be more relevant in a real-world policy setting: it may be infeasible to tailor the tax specifically to innovation prospects in the renewable fuel sector, if the tax is actually meant to address carbon use in the wider economy. We first note that if a mandate outperforms a naïve carbon tax, it must do so through its impact on innovation.

> **Remark 2.** If expected welfare under a mandate is greater than expected welfare under a naïve carbon tax, then the contribution to welfare due to innovation is greater under a mandate.

This follows from Remark 1 and equations (11) and (13). The contribution to welfare due to innovation is given by the term under the integrals in equations (11) and (13). These consist of the net profits of the innovator in both cases, as well as producer surplus for a carbon tax. For any given ω, whether net profits under a mandate are larger than net profits plus producer surplus under a carbon tax depends on $\bar{Q}$. In the next result, we show that a $\bar{Q}$ that induces R&D with the same probability as a carbon tax is not large enough for the mandate's net profits to exceed the carbon tax's net profits plus producer surplus. As a corollary, if a mandate ever achieves higher welfare than a naïve carbon tax, it must do so by inducing R&D more frequently than a carbon tax. In our model, that means inducing R&D even when technological opportunity is relatively low.

For the comparison that presumes the same R&D probability, we need to require that the innovation thresholds be the same under the two policies, i.e., $\hat{\omega}_m = \hat{\omega}_t$. Note that this (analytically convenient) criterion also equalizes the expected quality of realized innovations.

> **RESULT 2.** If a mandate is chosen so as to provide the same R&D incentive as the naïve carbon tax, then expected welfare is higher with the carbon tax.

The proof of this result starts by noting that Result 2 will hold so long as:

$$(14) \qquad S_0^* + \int_{\hat{\omega}_t}^{\varpi} \left\{ \frac{\omega^2}{12} + \frac{\omega^2}{24} - k \right\} dG(\omega) \geq S_0^m + \Pi_0^m - X_0^m + \int_{\hat{\omega}_m}^{\varpi} \left\{ \frac{\omega \bar{Q}}{2} - k \right\} dG(\omega)$$

Because, as noted in Remark 1, the given carbon tax achieves the first best (absent innovation) but the mandate does not, it must be that $S_0^* > S_0^m + \Pi_0^m - X_0^m$. Next, recall that the threshold under the naïve carbon tax $t = x$, given Condition 3, was shown to be $\hat{\omega}_t = \sqrt{12k}$, so that to ensure $\hat{\omega}_m = \hat{\omega}_t$ one needs $\bar{Q} = \sqrt{k/3}$. Hence, Result 2 will hold when the gains from innovation under the carbon tax exceed those under the mandate, i.e.,

$$(15) \qquad \int_{\hat{\omega}_t}^{\varpi} \left\{ \frac{\omega^2}{12} + \frac{\omega^2}{24} - k \right\} dG(\omega) \geq \int_{\hat{\omega}_t}^{\varpi} \left\{ \frac{\omega\sqrt{k/3}}{2} - k \right\} dG(\omega)$$

where the fact that the mandate is calibrated so that $\hat{\omega}_m = \hat{\omega}_t$ implies that the integrals in (14) have the same bounds. A sufficient condition for equation (15) to hold is that the integrand in the LHS exceed the integrand in the RHS for each ω. It is verified that the required condition is

(16) $\omega/4 \geq \sqrt{k/3} \quad , \quad \forall \omega \in [\hat{\omega}_t, \bar{\omega}]$

Because this condition is satisfied for the lower bound $\hat{\omega}_t = \sqrt{12k}$, and the LHS in (16) is increasing in ω, the condition is always satisfied.

Given the foregoing, Result 2 continues to hold when the mandate is calibrated so that the probability of R&D is lower than under a carbon tax. However, when mandates are chosen so that the probability of R&D is *higher* than under the carbon tax $t = x$, it is possible a mandate may achieve higher expected welfare than a naïve carbon tax. In such a case, the gain to welfare from inducing more innovation would need to be weighed against the costs of R&D and distortions to static welfare. We will return to this question in the numerical analysis section.

Competitive Innovation

Whereas the foregoing single-innovator setting is useful to fix ideas, it is a fact that in reality most industries feature multiple firms engaged in competing R&D projects. Modeling such a case is not trivial. A possibility is to presume a patent race contest: multiple agents compete for exactly the same innovation, and the first to invent obtains a patent that pre-empts all other innovators (Wright 1983). Because this setting results in a monopoly (only one patented innovation), it would simplify the analysis of post-innovation licensing. For the case of renewable energy of interest, however, we find it more appealing to presume that competing innovators are actually pursuing alternative innovation pathways which, if successful, can all be patented. Whether a (patented) innovation will be adopted in the marketplace is a different question, however, as it will depend on how good the innovation is relative to other realized innovations. To model this case we postulate the existence of a large number of potential innovators, and we assume there is free entry into the renewable energy innovation sector. Innovators are *ex ante* identical and observe a common technological opportunity signal ω. If they choose to conduct R&D, they obtain independent θ draws from $f(\theta|\omega)$. The innovator who draws the highest θ, denoted θ_1, has the best technology and becomes the exclusive licensor to the renewable energy production sector. However, as in Spulber (2013), the choice of royalty by the innovator who draws θ_1 is now constrained by the presence of competing innovators. Under Bertrand competition, the second-highest θ draw, denoted θ_2, is the binding constraint. Essentially, as compared with the foregoing analysis, θ_2 plays the same role as the pre-innovation production technique $\theta = 0$ for the single innovator case. But, of course, in the competitive innovation setting θ_2 is endogenous.

To characterize the pricing of innovation with multiple innovators, consider first the *laissez faire* setting. The innovator with the *ex post* best technology θ_1, presuming that $\theta_1 > \hat{\theta}$, sets the per-unit license r to maximize license profit, conditional on the competitive sector adoption, similar to the single innovator setting. But here the second best technology θ_2 may limit the price that the licensing innovator can extract. Specifically, the innovator with the best technology maximizes $r\left(c_1 - c_2 + \theta_1 - r\right)$, such that $r \leq \theta_1 - \theta_2$. For low realizations of θ_2, the constraint imposed by the second-best technology does not bind, the single innovator results continue to hold, and the solution is $r^* = Q_2 = (\theta_1 - \hat{\theta})/2$. Given this unconstrained royalty, it is apparent that the constraint $r \leq \theta_1 - \theta_2$ binds whenever $\theta_2 > (\theta_1 + \hat{\theta})/2$. In such a case the optimal royalty is $r^* = \theta_1 - \theta_2$, and $Q_2 = \theta_2 - \hat{\theta}$. The best innovator's maximum profit, denoted π_1, is therefore given by:

$$(17) \qquad \pi_1 = \begin{cases} (\theta_1 - \hat{\theta})^2/4 & \text{if } \theta_2 \leq (\theta_1 + \hat{\theta})/2 \\ \left(\theta_1 - \theta_2\right)\left(\theta_2 - \hat{\theta}\right) & \text{if } \theta_2 > (\theta_1 + \hat{\theta})/2 \end{cases}$$

The expected profit of a potential entrant now depends on the distribution of θ_1 and θ_2, which are best described by the concepts of "order statistics" widely used in auction theory (Krishna 2010). Specifically, given n innovators, the probability that an innovator's draw of θ is the maximum draw is equal to the probability that the $n-1$ other draws are smaller than θ. Because we have assumed a uniform distribution for the innovation projects, this probability equals $(\theta/\omega)^{n-1}$. Moreover, conditional on a given θ being the maximum draw, the second highest realization θ_2 is the maximum of $n-1$ independent draws from the uniform distribution on the support of $[0,\theta]$. Hence, the second highest realization θ_2 has cumulative distribution function $(\theta_2/\theta)^{n-1}$ and density function $\left((n-1)/\theta_2\right)\left(\theta_2/\theta\right)^{n-1}$. Using these results on the distribution of the first and second best innovations, we can determine the expected profitability of participating in the R&D contest. Specifically, with n entrants, the expected licensing profit of each innovator, given technological opportunity ω, can be written as:

$$\pi(\omega,n)=\int_{\hat{\theta}}^{\omega}\left\{\left(\frac{\theta_1+\hat{\theta}}{2\theta_1}\right)^{n-1}\frac{(\theta_1-\hat{\theta})^2}{4}+\int_{(\theta_1+\hat{\theta})/2}^{\theta_1}(\theta_1-\theta_2)(\theta_2-\hat{\theta})\frac{n-1}{\theta_2}\left(\frac{\theta_2}{\theta_1}\right)^{n-1}d\theta_2\right\}\left(\frac{\theta_1}{\omega}\right)^{n-1}\frac{1}{\omega}d\theta_1 \tag{18}$$

This term integrates over the range of values for θ that are both feasible and earn positive profit. Within the integral, profits are divided into two terms. When $\theta_2 \le (\theta_1+\hat{\theta})/2$, which occurs with probability $\left[(\theta_1+\hat{\theta})/2\theta_1\right]^{n-1}$, profit is given by the upper branch of equation (17). This is the first term under the integral. Conversely, whenever $\theta_2 > (\theta_1+\hat{\theta})/2$, profit is given by the lower branch of equation (17). This is captured by the second term, itself an integral over possible values of θ_2. Hence, equation (18) is the expected licensing profit when there is free entry under *laissez faire*.

Competitive Innovation Under a Carbon Tax

With the naïve carbon tax $t=x$, the innovator's problem is similar in structure to the *laissez faire* setting. But here, if the pre-innovation technology is such that Condition 2 applies, it is as if $\hat{\theta}=0$. Hence, given θ_1 and θ_2, the best innovator's licensing profit is:

$$\pi_t=\begin{cases}(\theta_1/2)^2 & \text{if } \theta_2\le\theta_1/2\\ (\theta_1-\theta_2)\theta_2 & \text{if } \theta_2>\theta_1/2\end{cases} \tag{19}$$

Given this conditional profit function, equation (18) can be adapted to yield the expected licensing profit $\pi_t(\omega,n)$ of each innovator facing technological opportunity ω when there are n innovators engaged in R&D:

$$\pi_t(\omega,n)=\int_0^{\omega}\left\{\left(\frac{1}{2}\right)^{n-1}\left(\frac{\theta_1}{2}\right)^2+\int_{\theta_1/2}^{\theta_1}\left(\frac{n-1}{\theta_2}\right)\left(\frac{\theta_2}{\theta_1}\right)^{n-1}(\theta_1-\theta_2)\theta_2 d\theta_2\right\}\left(\frac{\theta_1}{\omega}\right)^{n-1}\frac{1}{\omega}d\theta_1 \tag{20}$$

Performing the integration, and simplifying, yields:

$$\pi_t(\omega,n)=\frac{n-\left(1-(1/2)^n\right)}{n(n+1)(n+2)}\omega^2 \tag{21}$$

Note that when $n=1$ equation (21) reduces to $\omega^2/12$, which is what we found in equation (12) for the single innovator licensing profit. Profit is clearly increasing in technological opportunity ω. It is also verified that profit is decreasing in the number of innovators n (this occurs for two distinct reasons: as n increases, the probability of any one participant drawing the highest innovations decreases; and, as n increases, the expected royalty for any given innovation decreases).

The equilibrium number of innovators is determined by the zero profit entry condition. In equilibrium, noting that n is an integer, the number of innovators n_t^* satisfies:

$$\text{(22)} \qquad \pi_t\left(\omega, n_t^*\right) \geq k \geq \pi_t\left(\omega, n_t^* + 1\right)$$

To emphasize the dependence of the equilibrium number of firms on the R&D outlook parameter ω, which represents the asymmetric information between innovators and policy makers, in what follows this is denoted $n_t^* = n_t(\omega)$. From (22) it follows that equilibrium with free R&D entry and a carbon tax implies the existence of a sequence of thresholds $\hat{\omega}_t(n)$ such that there are *at least* n active innovators iff $\omega \geq \hat{\omega}_t(n)$. The threshold levels $\hat{\omega}_t(n)$ are readily computed from (21) and (22):

$$\text{(23)} \qquad \hat{\omega}_t(n) = \sqrt{\frac{n(n+1)(n+2)}{n-1+(1/2)^n}k}$$

Competitive Innovation Under a Mandate

Given a binding mandate $\bar{Q}$, an innovating firm in possession of the best technology θ_1 chooses the royalty rate to maximize $r\bar{Q}$, such that $c_2 - \theta_1 + \bar{Q} + r \leq c_2 - \theta_2 + \bar{Q}$. Clearly, the optimal royalty is $r^* = \theta_1 - \theta_2$ and the quantity induced is $\bar{Q}$. Therefore, the licensing profit of an innovator with the best technology θ_1, facing the second best technology θ_2, is $\pi_m = \left(\theta_1 - \theta_2\right)\bar{Q}$. Using the probability functions of the best and second-best innovations derived earlier, the expected profit of each entrant in the R&D contest, given n innovators and technological opportunity ω, is:

$$\text{(24)} \qquad \pi_m\left(\omega, n\right) = \int_0^{\omega}\left\{\int_0^{\theta_1}\frac{n-1}{\theta_2}\left(\frac{\theta_2}{\theta_1}\right)^{n-1}\left(\theta_1 - \theta_2\right)\bar{Q}d\theta_2\right\}\left(\frac{\theta_1}{\omega}\right)^{n-1}\frac{1}{\omega}d\theta_1$$

After integrating and simplifying, we obtain:

(25) $$\pi_m(\omega, n) = \frac{\bar{Q}}{n(n+1)}\omega$$

Expected profit is increasing in technological opportunity ω and the mandate $\bar{Q}$, and decreasing in the number of innovators. The equilibrium number of innovators $n_m^* = n_m(\omega)$ satisfies:

(26) $$\pi_m(\omega, n_m^*) \geq k \geq \pi_m(\omega, n_m^* + 1)$$

Similar to the case of the carbon tax discussed in the foregoing, equilibrium with free R&D entry and a mandate policy implies the existence of a sequence of thresholds $\hat{\omega}_m(n)$ such that there are *at least* n active innovators iff $\omega \geq \hat{\omega}_m(n)$. These threshold levels are computed from (25) and (26):

(27) $$\hat{\omega}_m(n) = \frac{n(n+1)}{\bar{Q}}k$$

Expected welfare is still the sum of consumer surplus, producer surplus, and licensing royalties less R&D costs and damages from the externality. [12] Assuming the mandate binds, producer surplus is unaffected by innovation. However, with competitive innovation, the consumer surplus and damages from the externality are impacted by innovation. In the single innovator case, the innovating firm appropriated all of the gains from innovation, so that the price of clean fuel was unchanged by innovation. Under competitive innovation, on the other hand, the winning innovator is only able to appropriate the gains to innovation stemming from improvements over the second best innovation. This means that the price of clean fuel falls by θ_2, which also reduces the blend price $\tilde{P}(Q)$ in equation (4). This leads to an expansion of energy consumption to a new equilibrium $Q(\theta_2)$ satisfying $\tilde{P}(Q(\theta_2)) = P(Q(\theta_2))$, where $Q(\theta_2) > Q(0)$ and $Q(0)$ is the pre-innovation equilibrium with mandates. Given a binding mandate, this demand expansion is met entirely by increased dirty fuel production. Whereas the price decline due to innovation raises consumer surplus, it also increases damages from externalities by $(Q(\theta_2) - Q(0))x$ and whenever $P(Q(\theta_2)) < c_1 + x$ each additional unit of dirty energy consumed reduces welfare.

Interestingly, therefore, innovation under a mandate contributes to a form of the so-called rebound effect. Prior to innovation a mandate raises the overall cost of energy and, similar to a carbon tax, reduces dirty energy consumption. But as innovation and our assumption of competitive licensing reduces the cost of renewable energy, the average cost of energy also falls, which leads to increased consumption of dirty energy. In some cases, this creates an incentive to curb innovation by reducing the mandate. Indeed, in our numerical simulations, we find that the stringency of the optimal mandate when there is free entry is sometimes lower than the optimal mandate when there is a single innovator.

Mandate vs. Carbon Tax with Competitive Innovation

With competitive innovation, the choice between a carbon tax and a mandate has a greater impact on the character of the realized innovation. To begin, it is more likely there will be at least n innovators under a carbon tax than under a mandate whenever $\hat{\omega}_m(n) \geq \hat{\omega}_t(n)$. By using equations (23) and (27), and simplifying, this condition reduces to:

$$(28) \qquad \frac{k}{\bar{Q}^2} \geq \frac{n+2}{\left(n-1+(1/2)^n\right)n(n+1)}$$

For any given policy the left hand side is fixed, while the right hand side is decreasing in n. This implies there is a threshold $\hat{n}$ such that $\hat{\omega}_m(n) \geq \hat{\omega}_t(n)$ whenever $n > \hat{n}$, where $\hat{n}$ is defined by:

$$(29) \qquad \frac{\hat{n}+1}{\left(\hat{n}-2+(1/2)^{\hat{n}-1}\right)(\hat{n}-1)\hat{n}} \geq \frac{k}{\bar{Q}^2} \geq \frac{\hat{n}+2}{\left(\hat{n}-1+(1/2)^{\hat{n}}\right)\hat{n}(\hat{n}+1)}$$

Because $\hat{\omega}_m(n) \geq \hat{\omega}_t(n)$ for all $n \geq \hat{n}$, and given that $\hat{\omega}_m(n)$ and $\hat{\omega}_t(n)$ are monotonically increasing in n, we conclude with the following result.

> **RESULT 3.** Whenever technological opportunity exceeds a certain threshold, i.e., $\omega \geq \hat{\omega}_t(\hat{n})$, the number of innovators is (weakly) higher under a carbon tax than under a mandate. Conversely, whenever $\omega \leq \hat{\omega}_t(\hat{n})$, the number of innovators is (weakly) higher under a mandate policy than a carbon tax.

Under either policy, the realized innovation is the best technology drawn by any of the innovators, denoted θ_1. Conditional on the technology opportunity parameter ω and the number of innovators n, the expected new technology is

(30) $$E[\theta_1|n,\omega] = \int_0^{\omega} \theta f_1(\theta|n,\omega)d\theta$$

where $f_1(\theta|n,\omega)$ here is the density function of the distribution of the highest order statistics, which can be related to the primitive distribution $f(\theta|\omega)$ (Krishna 2010). Because of our assumed uniform distribution $f(\theta|\omega) = 1/\omega$, it follows that

(31) $$f_1(\theta|n,\omega) = n\left(\frac{\theta}{\omega}\right)^{n-1}\frac{1}{\omega}$$

Using this density function and performing the integration in (30) we find:

(32) $$E[\theta_1|n,\omega] = \frac{n}{n+1}\omega$$

Of course, as discussed in the foregoing, the equilibrium number of innovators will depend on the actual technology opportunity ω and on the policy in place, i.e., $n = n_i(\omega)$, $i = t, m$. Furthermore, from the perspective of a regulator (who does not observe ω), what is relevant is the expectation of the best technology conditional only on the choice of policy, that is

(33) $$E[\theta_1|i] = \int_0^{\varpi} \frac{n_i(\omega)}{n_i(\omega)+1}\omega dG(\omega)$$

This makes it apparent that, given the primitive distribution of technological opportunities $G(\omega)$, the expected technology realized depends only on the number of innovators induced by the policy $i = t, m$ for every opportunity ω.

Earlier, we showed that, for the single innovator case, setting a mandate equal to $\bar{Q} = \sqrt{k/3}$ ensures that R&D occurs under either policy with equal probability. When $\bar{Q} = \sqrt{k/3}$, then equation (29) is satisfied by $\hat{n} = 1$ and $n_t(\omega) \geq n_m(\omega)$ for all ω. By equation (33) this implies the expected technology in use will be higher under a carbon tax.

> **RESULT 4.** When the mandate is such that the probability of R&D under a mandate is equal to the probability of R&D under a carbon tax, then the expected technology realized after innovation is better under a carbon tax.

What if the mandate $\bar{Q}$ were tuned so that the expected best technology is the same as under the carbon tax? In order for $E[\theta_1]$ to be the same under either policy, the mandate must be increased from $\sqrt{k/3}$, so that $\hat{\omega}_m(n)$ is decreased. Because $\hat{\omega}_m(n) = n(n+1)k/\bar{Q}$, increasing $\bar{Q}$ will decrease $\hat{\omega}_m(n)$ for all n. Specifically, we will now have $\hat{\omega}_m(1) < \hat{\omega}_t(1)$ so that R&D is more likely to occur under a mandate than under a carbon tax. Moreover, for $E[\theta_1]$ to be the same under either policy, it cannot be that $n_m(\bar{\omega}) > n_t(\bar{\omega})$, where $n_i(\bar{\omega})$ is the number of innovators under policy i and the best possible technological opportunity. If this were the case, then by Result 3, $n_m(\omega) > n_t(\omega)$ for all $\omega \in [0, \bar{\omega}]$ and by equation (33) $E[\theta_1]$ would be higher under a mandate. Therefore, in this setting, there is some intermediate threshold $\hat{n}$, satisfying $1 < \hat{n} < n_m(\bar{\omega})$, where the number of innovators is higher under a carbon tax for $\omega \geq \hat{\omega}_t(\hat{n})$ and higher under a mandate otherwise. This implies:

> **RESULT 5.** When the mandate is such that the expected best technology is the same under either policy, then the distribution of outcomes under a carbon tax is more disperse than under a mandate.

Under a carbon tax there is a higher probability of a very good innovation or none at all. A mandate has a higher probability of *some* innovation, but a lower probability of a very good innovation, since it produces weaker incentives to innovate when technological opportunity is very high.

Numerical Analysis

The foregoing analysis has provided some interesting qualitative results on the comparison between mandates and the alternative of a carbon tax. While these results are illuminating, a limitation is that, apart from Results 2, not much has been said about welfare effects. This is not surprising because specific welfare conclusions should depend on the particular shape of the demand function $P(Q)$ and on the distribution of technological opportunities $G(\omega)$. Also, our analytical results have been contingent on a few assumptions: that clean energy is on the cusp of being competitive with (taxed) fossil fuels (Condition 3), and that the mandate is always binding (Condition 2). In this section we relax these conditions and specify explicit functional forms for $P(Q)$ and $G(\omega)$ so that we may consider the impacts of the policy instruments of interests in a more general context by means of a numerical analysis.

Parameterization

We begin by normalizing $c_1 = 100$, so that a tax on dirty energy can be interpreted as a percent of the *laissez-faire* price level. In the baseline parameterization the externality is calibrated to $x = 20$, so that it amounts to 20% of the private cost of dirty energy,[13] and we put $c_2 = 120$, consistent with Condition 3 (but this condition does not hold when the marginal damage x is changed from its baseline value). Next, we postulate the inverse demand function $p(Q) = (a - \ln Q)/b$ or, equivalently, that the direct demand function for energy takes the semi-log form:

(34) $$\ln Q = a - bp$$

This is a convenient parameterization which, among other desirable features, can accommodate various hypotheses concerning demand elasticity $\eta \equiv -\partial \ln Q / \partial \ln p$. For this function $\eta = bp$, hence the parameter b can be varied to implement alternative elasticity values. The parameter a is calibrated so that total demand for energy at price $p = c_1$ (and at the baseline elasticity value) is equal to $Q = 100$, that is we put $a = bc_1 + \ln 100$. This normalization means that we can interpret the level of mandates as the percent of total demand under a *laissez-faire* policy. As for $G(\omega)$, we assume that ω is distributed on $[0, \bar{\omega}]$ by an appropriately scaled beta distribution. The probability density function $g(\omega)$ is therefore given by:

(35) $$g(\omega; \alpha, \beta) \propto (\omega / \bar{\omega})^{\alpha-1} (1 - \omega / \bar{\omega})^{\beta-1}$$

where the parameters α and β determine the moments of this distribution and govern its shape. This distribution is very flexible, and alternative choices of α and β can yield both symmetric and skewed density functions. We normalize $\bar{\omega} = 120$ so that, under all possible innovation, the marginal cost of clean energy remains non-negative everywhere.

Given the foregoing functional form assumptions and parametric normalizations, we still have four free parameters that can be varied to gain some insights in the nature of the results. The first of these is the elasticity of demand η. Because this value depends on the evaluation price, for clarity we will always measure elasticity with reference to the *laissez-faire* price of energy, where $p = c_1$. For our baseline, we set b so that $\eta = 0.5$. We also consider the cases where $\eta = 0.25$ and $\eta = 1$ (these values reflect the widely-held belief that energy demand is inelastic; see Toman, Griffin and Lempert 2008, p. 18). Second, we vary the cost of the externality x. As noted, for the baseline we set $x = 20$, but we also consider the cases of $x = 10$ and $x = 40$. Third, we vary the R&D cost k. To calibrate this parameter we relate it to the magnitude of profits that innovation can produce in the *laissez-faire* baseline. Under the highest level of technological opportunity, the expected profit for a single innovator, in view of (10) and the chosen normalizations, is equal to $\pi_t(\bar{\omega}) = 6,250/9$. We consider values of k equal to 3%, 6%, and 12% of this profit level, with 6% corresponding to the baseline.

Fourth, we vary the shape of the distribution of technological opportunity $G(\omega)$. The first moment of the assumed beta distribution is $E[\omega] = \bar{\omega}\alpha/(\alpha+\beta)$. We set $\alpha + \beta = 2$ and, by varying the parameters α and β, we obtain both different values for $E[\omega]$ and different shapes. The baseline parameters are $\alpha = 0.5$ and $\beta = 1.5$, which yield $E[\omega] = 30$. This is a positively skewed distribution (low draws of ω are more likely than high ones), which reflects the belief that technological opportunity is more likely to be consistent with incremental innovation than major breakthroughs. The other two cases we consider are $\alpha = 0.25$ and $\beta = 1.75$, which yield $E[\omega] = 15$ (and correspond to an even more positively skewed distribution), and $\alpha = 1$ and $\beta = 1$, which yield $E[\omega] = 60$ (and correspond to a uniform distribution where high draws of ω are equally likely as low ones). As for the policies t and $\bar{Q}$, for each set of parameters we numerically solve for the value of the policy instrument that maximizes welfare, i.e., expected Marshallian surplus (all calculation are coded in Matlab). More details on these calculations are available in the Supplementary Appendix available online.

As noted, the optimal mandates and carbon taxes thus computed are second best policies. It is of some interest to see how they compare with first best allocations. To compute the latter, we assume the social planner can directly choose the number of innovators, upon observing the technological opportunity draw ω, and also choose the energy quantities Q_1 and Q_2 upon learning the best realized technology θ_1. Optimal energy use equalizes the marginal social cost of each form of energy with price, i.e., $c_2 - \theta_1 + Q_2 = c_1 + x = P(Q_1 + Q_2)$. Note that here welfare depends on θ_1 alone, which we denote $W(\theta_1)$. Working backward, the optimal number of innovating firms solves

(36) $$\max_{n}\left\{\int_0^{\omega} W(\theta) f_1(\theta | n,\omega) d\theta - nk\right\}$$

If $E\left[W^* | \omega\right]$ denotes the value function of the program in (36), the expected value of first best welfare is given by:

(37) $$E\left[W^*\right] = \int_0^{\bar{\omega}} E\left[W^* | \omega\right] dG(\omega)$$

Results

The experiments we report, as described in the foregoing, encompass $3^4 = 81$ different parameter combinations. Some basic descriptive results for the baseline parameters are reported in Table 1. For the single innovator case the expected number of innovators $E[n]$ can be interpreted as the probability that R&D will be conducted. In the baseline setting, under a *laissez-faire* policy, R&D is conducted with probability 0.25 for the single innovator case. The expected quality of innovation $E[\theta_1]$ is 9.58, which improves to 15.93 with competitive innovation. Hence, in either case the "average" technology under *laissez faire* is insufficient to compete with fossil fuels (the minimum inventive step here is $\hat{\theta} = 20$). Still, some innovation does take place under *laissez-faire*, because some better-than-average draws are viable. The expected quantity of clean energy consumed is small but not negligible, at 2.60 and 8.75 under the single innovator and competitive innovation cases respectively (recall that the *laissez-faire* quantity of total energy consumed was normalized to 100).

An optimal policy (mandate or tax) raises all these quantities, and also improves welfare. The expected quality of innovation $E[\theta_1]$, as well as the expected quantity of clean energy produced $E[Q_2]$, is significantly increased for mandates and the carbon tax. Under an optimal mandate, the probability of R&D more than triples, relative to the *laissez-faire* case, and the expected number of innovators, given competitive innovation, increases from 1.52 to 2.66. Compared with the carbon tax, the mandate induces a greater probability of innovation with a single innovator, but a carbon tax has a higher expected number of entrants when there is competitive innovation. As discussed earlier, this is because a mandate provides comparatively strong incentives to conduct R&D when technological opportunity is low, and this induces firms to enter for more draws of ω than under a carbon tax. The expected profit of R&D increases as ω rises, but it increases at a faster rate for the carbon tax. In the single innovator case, this is irrelevant, since the firm makes a binary decision to conduct R&D or not. But in the competitive innovation case, the higher profits of a carbon tax can support more innovators, and this leads to a higher overall expected number of entrants (3.08 in a carbon tax, compared to 2.66 under a mandate).

The expected quality of innovation, however, is highest under a mandate in each case (and in fact, higher than the first-best). In the competitive innovation case, this stems from the differential impact of entrants. Consistent with Result 5, we note that carbon taxes will tend to have more disperse results than the mandate, inducing either many innovators or none at all. Because $\partial^2 E[\theta_1|\omega,n]/\partial n^2 < 0$, the marginal impact of additional entrants under a tax when ω is high (and there are already many firms) is lower than that of additional entrants under a mandate when ω is low (and there are few or no entrants).

The expected quantity of clean energy produced is higher under a mandate, when there is a single innovator, but higher under a carbon tax in the competitive innovation case. However, in both cases, welfare is highest under an optimal carbon tax.[14] In fact, this is part of a general numerical result, and we have found it to be true beyond the baseline.

> **RESULT 6 (NUMERICAL).** In all parametric cases considered: (a) For both competitive innovation and single innovator cases, expected welfare under the optimal mandate is always lower than under the optimal carbon tax. (b) For the competitive innovation case, expected welfare under the optimal mandate is always lower than under the naïve carbon tax.

Result 6 refers to 81 different parameter combinations, each of which is solved under single innovator and competitive innovation conditions. This result suggests that an optimal mandate, while it improves welfare relative to *laissez-faire*, is inferior to an optimal carbon tax. Upon comparing the outcomes associated with the optimal policy tools with first best allocations, we note the importance of allowing for an endogenous number of innovators, as done in this paper. The single innovator case fares poorly vis-à-vis the first best, whereas the outcomes associated with competitive innovation are fairly close to the first best (especially for the carbon tax).

To gain further insights into the comparison of the policy instruments being considered, Table 2 illustrates the sensitivity of optimal policies to changes in the calibrated parameters. The first row reiterates the optimal policies for the baseline parameterization reported in Table 1. Each subsequent row presumes the same parameters as the baseline, except along one dimension. For example, in the second row the elasticity of demand, evaluated at the *laissez-faire* price, is changed to $\eta = 0.25$. Each column gives the optimal policy value across different policy instruments and assumptions about innovation. Looking at the first six columns, it is apparent the optimal mandate is much more profoundly affected by the presence or absence of innovation than the optimal carbon tax. This suggests the optimal choice of a mandate is sensitive to information about the innovation context, about which policy makers might be less informed than innovators. This conclusion is buttressed by the last two lines of Table 2, which give the optimal policies when the outlook for technological innovation is altered. For example, if this outlook improves from $E[\omega] = 30$ to $E[\omega] = 60$, the optimal mandate increases by 96% in the competitive innovation case, whereas the corresponding optimal carbon tax increases only by 6%. Note also that, consistent with Result 1, the stringency of the optimal mandate in the presence of a single innovator is always (weakly) greater than the optimal mandate without innovation. However, because of the rebound effect under a mandate when there is competitive licensing, the result does not carry through to the free entry case. In several parameter combinations considered, the stringency of the optimal mandate is reduced when we move from a single innovator to the competitive innovation case (although the competitive innovation mandate remains larger than the no innovation mandate).

In view of the fact, illustrated in Table 2, that the optimal carbon tax is less sensitive to the innovation context than the optimal mandate, we also compared the performance of the naïve carbon tax $t = x$ (which, strictly speaking, is optimal only absent the prospect of innovation) with the optimal mandate. This comparison is of some interest, in an applied policy context, because the information requirement to compute this tax level is clearly much lower than required by the optimal instruments. As noted in Result 6 (b), it turns out that, for the competitive innovation case, even the naïve carbon tax dominates the optimal mandate in terms of welfare.[15]

Whereas Table 2 illustrates that the magnitude of an optimal policy is more sensitive to information about innovation under a mandate than under a carbon tax, Table 3 shows that welfare outcomes are also more sensitive under a mandate. Specifically, in Table 3, for each policy tool, we decompose the total welfare change $W_1^* - W_0^0$, where W_1^* is the expected welfare with innovation under the optimal instrument choice (mandate or carbon tax, all for the competitive innovation case), and W_0^0 is welfare under *laissez-faire* and no innovation. The decomposition identifies the following four additive components: (i) $W_1^0 - W_0^0$, the expected welfare due to innovation under *laissez-faire;* (ii) $W_0^n - W_0^0$, the "static" gain in expected welfare, due to the reduction of the externality, with a "naïve" level of the policy instrument (i.e., one that does not account for the prospect of innovation); (iii) $(W_1^n - W_0^n) - (W_1^0 - W_0^0)$, the additional gain in expected welfare, relative to *laissez-faire,* due to *policy-supported* innovation (with a naïve level of the instrument); and (iv) $W_1^* - W_1^n$, the additional gain in expected welfare from moving to an optimal level of the policy instrument. Finally, the last column reports the expected welfare associated with the first-best solution, also compared with to the pre-innovation *laissez-faire* case, $W^* - W_0^0$.

We find that the terms in (i)—the gain in expected welfare from innovation in a *laissez-faire* setting—represent the largest components of this decomposition. This feature is of some interest *per se*, as it emphasizes that the market mechanisms that rationalize the use of policy instruments to spur innovation also work, to a degree, when no such support is present. Under a carbon tax, the components in (ii) and (iii) dominate (iv). By contrast, under a mandate, most of the gain in welfare, relative to *laissez faire*, is associated with (iv) (the terms in (ii) and (iii) are very small for all parametric combinations we considered). That is, under a mandate it really is important to tune the policy instrument in response to innovation, whereas with a carbon tax most of the welfare gain can be achieved with the naïve (static) level of the policy instrument. Indeed, as can be seen by comparison with the last column, both a naïve and optimal carbon tax get quite close to the expected welfare for the first-best allocation.

Conclusion

The direct impact of most environmental policy tools is to steer the economy's resources away from polluting activities and towards cleaner ones. This reallocation of resources, in addition to ameliorating the externality effect from a static perspective, also has important dynamic implications because it creates R&D incentives (the induced innovation hypothesis). In this paper we have studied these issues for the case of "mandates" that establishes minimum use quotas for certain goods, a policy tool that is central to U.S. biofuel policies and that is becoming increasing popular in renewable energy contexts. We find that mandates can in fact improve upon *laissez faire*, and that, with a single innovator, the prospect of innovation increases the optimal mandate level. In a competitive R&D setting, however, it is possible that renewable energy innovation contributes to an increase in the consumption of fossil fuels, and so it may be desirable to reduce the mandate relative to the single innovator case. With mandates, the innovation effects are critical and account for most of the desirable welfare impacts of this policy tool. Our numerical results, however, indicate that an optimally calibrated mandate is much more sensitive than the optimal carbon tax to assumptions about the innovation process, such as the nature of competition in innovation and the outlook for technological opportunity. In general, the more promising is the outlook for innovation, the higher the mandate ought to be. Indeed, the optimal mandate is such that it would typically induce welfare losses in the *status quo* without innovation. In any event, we find that the optimal mandate policy, although it is better than *laissez faire*, is clearly dominated by a carbon tax policy.

Our numerical results show that the contribution of clean energy innovation to welfare is large relative to the static impact of reallocation that environmental policies bring about, highlighting the importance of designing policies with innovation in mind. Our analysis also shows that market-based incentives are conducive to innovation even for the *laissez faire* scenario. Indirectly, this is a reminder that innovation is not a prerogative of renewable energy and clean technologies only: market-led innovation in polluting sectors is equally likely, as recent experience with breakthroughs in natural gas and shale oil extraction indicate. If anything, this consideration provides greater urgency for effective policies supporting pollution reduction and clean energy development: the longer renewable energy innovation is underprovided, the more difficult is the task of developing clean energy capable of competing with dirty energy (a point emphasized in Acemoglu et al. 2012).

A novel contribution of our paper, stemming from the explicit stochastic innovation framework that we have developed, is to shed some light on the extent to which alternative policies matter for the distribution of the quality of innovation. In our setting, innovators observe a signal on the actual innovation prospects before making their R&D investment. Compared with a mandate, a carbon tax tends to create high profit opportunities when the outlook for R&D turns out to be very good, which induces a flurry of activity that makes the realization of the good innovation outcome likely. Conversely, when the outlook for R&D is weak, mandates may provide more incentive for innovation. Hence, mandates may be a useful policy tool to incentivize R&D when only minor innovations are attainable, or when the problem at hand is simply to promote adoption of existing technologies, as for first-generation biofuels (e.g., the case of corn-based ethanol mentioned in the introduction). But when the goal is to promote breakthrough innovations, as for the case of second-generation (advanced) biofuels, a carbon tax is preferable to mandates. The analytical and numerical results of our model are therefore quite supportive of the general perspective articulated by Arrow et al. (2009) and discussed in the introduction. We also note that our qualitative conclusions appear consistent with an emerging empirical literature in renewable energy which shows that quantity-based policies have positive and statistically significant predictors of innovation *only* for older technologies, whereas price-based policies have positive and statistically significant impact for younger technologies (Johnstone, Hascic and Popp 2010).

Whereas mandates may be of limited effectiveness at spurring innovation for breakthrough technologies, their superior ability to induce innovation when incremental innovation is more likely may make them desirable in some settings. For example, if learning-by-doing is believed to be an important source of technological advance in a field, then it may be more desirable to guarantee that there is *some* kind of innovation, even if it is of low quality, so that the dynamics of learning-by-doing can get started. Alternatively, when innovation proceeds in many incremental steps, mandates may provide higher incentives than a carbon tax for each step in isolation. On the other hand, knowledge spillovers that raise the productivity of R&D in competitive settings (which we have neglected in our model) may strengthen one of our results: because a carbon tax features more innovators under favorable technological opportunity, spillovers may further increase the dispersion of realized innovation. Notwithstanding these qualifications, the general conclusion is that a mandate policy is not a very effective tool to promote market-based innovation. Indeed, for the general case of an endogenous number of multiple innovators, we find that even the naïve carbon tax, which ignores the prospect of innovation, yields higher expected welfare than an optimal mandate.

Figure 1. Timeline of information and actions

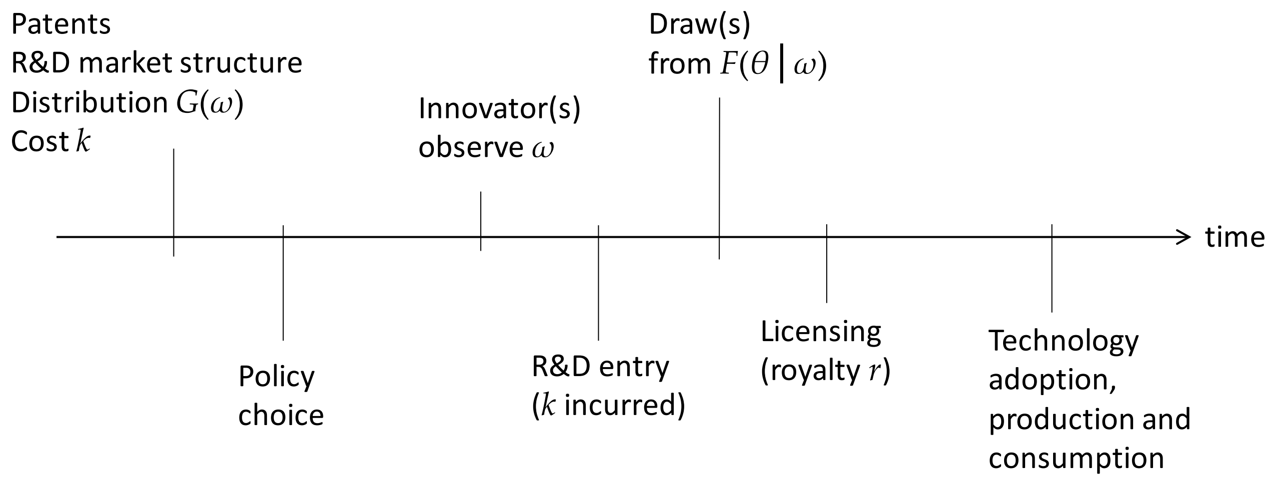

Figure 2. Conventional and renewable energy: Innovation and production costs

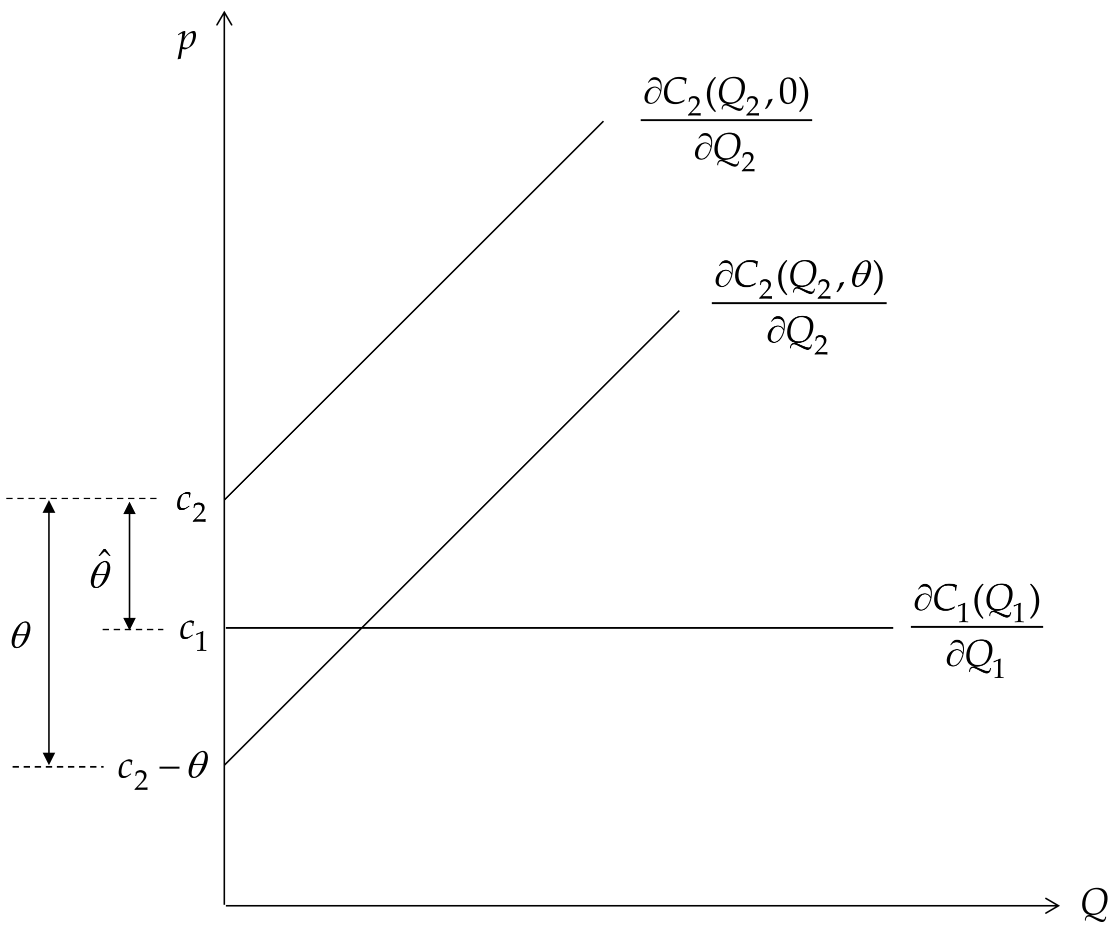

Figure 3. Feasible and unfeasible mandates

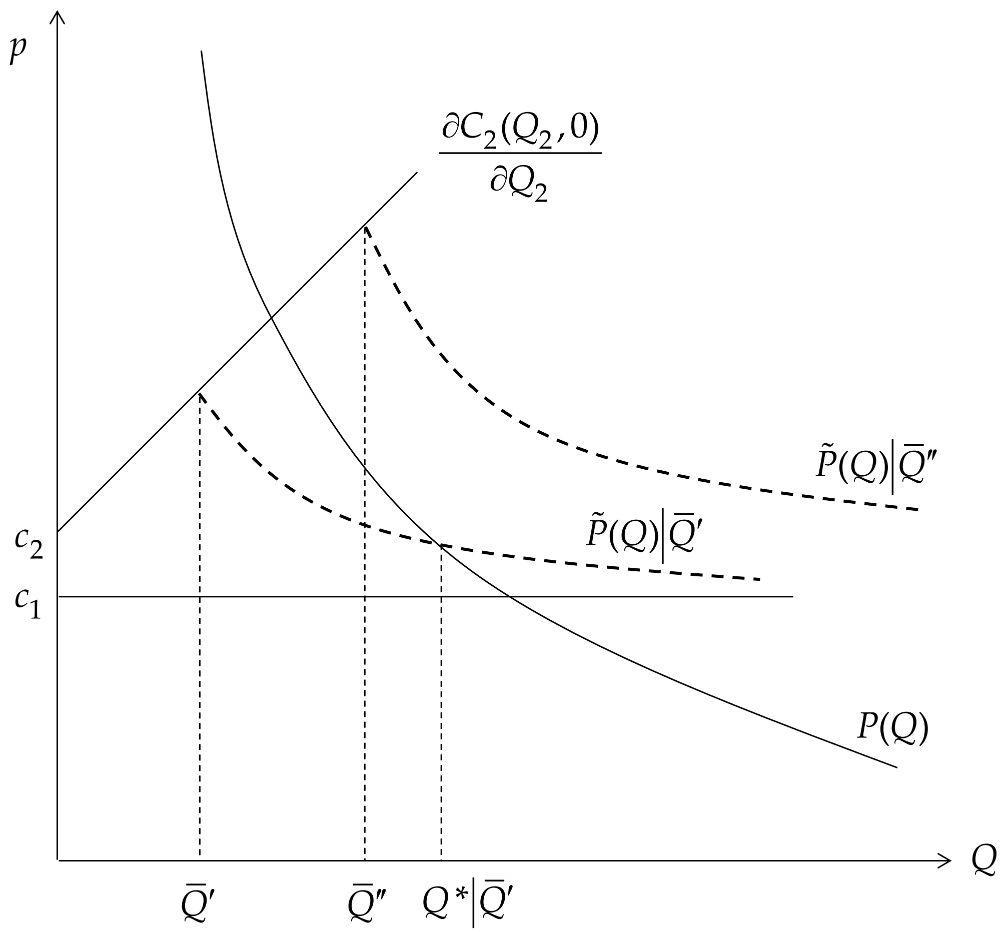

Table 1. Numerical Results for Baseline

	Single Innovator			Competitive Innovation			
	Laissez-Faire	Mandate	Carbon Tax	*Laissez-Faire*	Mandate	Carbon Tax	First Best
Optimal instrument	-	18.22	23.41	-	16.00	23.40	-
$E[n]$	0.25	0.78	0.56	1.52	2.66	3.08	3.33
$\sqrt{Var(n)}$	0.44	0.42	0.50	3.10	2.83	3.95	4.19
$E[\theta_1]$	9.58	15.39	14.28	15.93	24.43	23.91	24.33
$\sqrt{Var(\theta_1)}$	20.43	19.67	20.29	29.94	28.00	29.74	29.76
$E[Q_2]$	2.60	18.23	9.64	8.75	21.44	22.93	24.07
$\sqrt{Var(Q_2)}$	6.91	0.63	9.60	18.96	13.68	26.63	29.08
$E[W]$	123	141	310	402	442	676	695
$\sqrt{Var(W)}$	414	360	537	977	974	1,098	1,103

Note: the baseline parameters are $\eta = 0.5$, $x = 0.2\, c_1$, $k = 0.06\, \pi(\varpi)$, $\alpha = 0.5$ and $\beta = 1.5$.

Table 2. Optimal Policy Instruments under Alternative Assumptions

	Optimal Mandate			Optimal Carbon Tax		
	No Innovation	Single Innovator	Competitive Innovation	No Innovation	Single Innovator	Competitive Innovation
Baseline	2.4	18.2	16.0	20.0	23.4	23.4
$\eta = 0.25$	1.1	1.2	13.8	20.0	24.3	23.4
$\eta = 1$	5.2	15.0	15.5	20.0	22.5	22.7
$x = 10$	0.0*	0.0*	0.0*	10.0	13.9	14.4
$x = 40$	30.3	41.7	46.2	40.0	47.4	42.9
$k = 0.03\overline{\pi}$	2.4	19.1	15.6	20.0	23.9	22.3
$k = 0.12\overline{\pi}$	2.4	18.2	16.0	20.0	23.0	24.0
$E[\omega] = 15$	2.4	9.1	10.0	20.0	21.7	21.8
$E[\omega] = 60$	2.4	31.2	31.3	20.0	29.2	24.8

Note: Each row changes one parameter, all other parameters as in the baseline. * reflects rounding (optimal mandates are strictly positive).

Table 3. Welfare Decomposition under Alternative Assumptions (Competitive Innovation)

	Baseline	$\eta = 0.25$	$\eta = 1$	$x = 10$	$x = 40$	$k = 0.03\bar{\pi}$	$k = 0.12\bar{\pi}$	$E[\omega] = 15$	$E[\omega] = 60$
First best	**695**	**648**	**783**	**419**	**1,662**	**781**	**589**	**384**	**1,788**
Optimal Mandates	**442**	**427**	**474**	**315**	**1,275**	**548**	**312**	**190**	**1,440**
(i) *laissez faire* innovation	402	402	402	315	577	504	281	174	1,335
(ii) static gain, naïve mandate	*2*	*1*	*12*	*0*	*400*	*2*	*2*	*2*	*2*
(iii) policy-induced innovation, naïve mandate	*3*	*2*	*13*	*0*	*173*	*5*	*4*	*2*	*3*
(iv) naïve to optimal mandate	*34*	*22*	*47*	*0*	*125*	*36*	*25*	*11*	*99*
Optimal Tax	**676**	**630**	**767**	**403**	**1,649**	**770**	**556**	**370**	**1,763**
(i) *laissez faire* innovation	402	402	402	315	577	504	281	174	1335
(ii) static gain, naïve tax	*97*	*49*	*187*	*25*	*575*	*97*	*97*	*97*	*97*
(iii) policy-induced innovation, naïve tax	*170*	*171*	*168*	*55*	*490*	*165*	*169*	*97*	*317*
(iv) naïve to optimal tax	*7*	*8*	*5*	*8*	*7*	*4*	*9*	*3*	*14*

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Footnotes

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[1] Another prominent example of this kind of policy is given by renewable portfolio standards which mandate that suppliers of electricity source a set percentage of electricity from renewable sources such as solar, wind, biomass, and hydroelectric providers (Holland 2012). As of 2011 they were used in 27 US states (Delmas and Montes-Sancho 2011), and six European countries (Haas et al. 2011).

[2] de Gorter, Drabik and Just (2015) provide a useful introduction to previous work and discuss its main themes, which have emphasized the impact of expanded biofuel production on commodity prices, welfare and the environment, including the controversial issue of indirect land use changes.

[3] Jaffe, Newell and Stavins (2003), Requate (2005), and Popp, Newell and Jaffe's (2010) provide comprehensive reviews.

[4] A portfolio of instruments is bound to outperform any individual policy in our second-best setting (Fischer, Parry and Pizer 2003, Fischer and Newell 2008). Nonetheless, here we compare individual policy tools because our objective is to clarify the effectiveness of mandates as R&D incentives.

[5] Biglaiser and Horowitz (1994), Parry (1995), and Laffont and Tirole (1996) are notable exceptions.

[6] Indeed, for environmental innovations where the underlying externality is not fully internalized by private agents, this under-provision problem is believed to be most acute (Jaffe, Newell and Stavins 2005).

[7] Thus, as in Denicolo (1999), Laffont and Tirole (1996), Scotchmer (2011), and Acemoglu et al. (2012), we model innovation as a replacement technology (rather than an abatement technology).

[8] In the Supplementary Appendix, available online, we also investigate the possibility that an innovator may choose multiple independent research projects and show that close analogues for the results of this paper hold.

[9] Thus, we model the effect of innovation as a parallel downward shift in the marginal cost curve of renewable energy. More generally, both intercept and slopes may be affected. Restricting attention to the structure in (3), however, is extremely convenient for the tractability of the analysis we present.

[10] Since Arrow (1962), the innovator's inability to fully appropriate the social value of the innovation has been recognized as an important feature of R&D activities. Postulating that licensing takes place via a per-unit royalty is meant to capture this feature. In the Supplementary Appendix available online, however, we also consider a two-part tariff and find that our results are robust to this more general licensing framework.

[11] US biofuel mandates provide an example of effective enforcement. Such mandates, as per the EISA legislation, are specified in total volume terms. Enforcement then relies on fractional requirements set annually by the U.S. Environmental Protection Agency (EPA) and imposed on obligated parties (Schnepf and Yacobucci 2013, Lade and Lin Lawell 2016). When exogenous contingencies change from year to year, the EPA is expected to adjust such fractional requirements to ensure that the statutory total volume mandates are met.

[12] Complete rent dissipation does not occur, despite free entry, because the number of innovators is (accurately) modeled as an integer. We provide further detail on computing expected welfare in the Supplemental Appendix.

[13] This value for the externality cost is meant to be somewhat representative of estimates for the social cost of carbon relative to the cost of transportation fuel. The US government's estimate for the 2015 social cost of carbon, in 2007 dollars, is $37/ton of CO2 if a 3% discount rate is used, and $57/ton of CO2 if a 2.5% discount rate is used (U.S. Government 2013, p. 3). These discount rates have been criticized for being too high (Johnson and Hope 2012), and so we use the figure associated with the lower 2.5% discount rate as our baseline. Converting this estimate to 2015 dollars yields a social cost of $65/ton of CO2. The carbon emission coefficient is 8.9 kg CO2/gallon of gasoline (U.S. EPA 2014), which implies a social cost of carbon is $0.58 per gallon. Taking the benchmark price of gasoline to be $3.00/gallon, then the damage imposed by the carbon externality is approximately 20% of the cost of fuel, which is reflected in our baseline value of $x = 20$.

[14] Throughout, welfare is measure as expected Marshallian surplus, normalized to zero at the pre-innovation, *laissez-faire* case.

[15] In the single innovator case, expected welfare was higher with the optimal mandate than with the naïve carbon tax for 9 of the 81 parameters combinations that we considered.

Title: AJAE Appendix for "**Mandates and the Incentive for Environmental Innovation**"

Authors: **Matthew S. Clancy** and **GianCarlo Moschini**

Date: 16 July 2017

Note: The material contained herein is supplementary to the article named in the title and published in the *American Journal of Agricultural Economics* (AJAE).

TABLE OF CONTENT

APPENDIX A. TWO-PART TARIFF LICENSING

The model in the main text presumes a simple licensing structure whereby the successful innovator charges a per-unit royalty r to producers in the renewable industry. In this Appendix we explore the robustness of the results reported in the paper to a more general licensing structure. Specifically, here we consider a two-part tariff scheme, such that the innovator charges a per-unit royalty $r \geq 0$ as well as a fixed fee $F \geq 0$. Moreover, we explicitly allow the innovator to ration licenses. We show that the paper's results are robust to this change of licensing structure, provided the mandate is sufficiently stringent.

To consider the licensing problem of the innovator in more detail, it is helpful to represent the postulated upward-sloping industry supply curve for renewable energy production in terms of individual firms' supply schedules. The formulation that we discuss in this appendix considers an industry made up of N identical price-taking production firms. To yield the industry marginal cost structure postulated in the paper, each of these firms has marginal production costs $mc = c_2 + Nq$, where we have chosen the units of q to ensure a firm-level marginal cost slope of N. For a given output price p, the firm-level supply curves of these firms are:

$$q = \frac{(p - c_2)}{N}$$

Note that, when summed over N firms, this delivers the paper's aggregate supply curve $Q = p - c_2$.

Here we consider explicitly the more general case of competitive innovation, where licensing of the best innovation θ_1 is constrained by the availability of the second-best innovation θ_2. The Bertrand competition framework discussed in the text ensures that the latter is freely available. Hence, provided $p \geq c_2 - \theta_2$, a competitive firm in the renewable energy industry using the second-best technology would produce quantity

$$q_2 = \frac{(p - c_2 + \theta_2)}{N}$$

and earn net returns (quasi rent) of $R_2 \equiv pq_2 - \left[(c_2 - \theta_2)q_2 + Nq_2^2/2 \right]$, that is:

$$R_2 = \frac{\left(p - c_2 + \theta_2\right)^2}{2N}$$

As for the best technology θ_1, under the two-part tariff scheme with a per-unit royalty $r \geq 0$ and a fixed fee $F \geq 0$, a firm that licenses this technology would produce quantity

$$q_1 = \frac{\left(p - c_2 - r + \theta_1\right)}{N}$$

and earn net returns

$$R_1 = \frac{\left(p - c_2 - r + \theta_1\right)^2}{2N} - F$$

Furthermore, we allow the innovator to possibly restrict the availability of this technology by issuing $\tilde{N} \leq N$ licenses. Given that, total industry production is $Q = \tilde{N}q_1 + (N - \tilde{N})q_2$. Hence, for a given output price p the industry supply curve is:

$$Q = \frac{\tilde{N}}{N}\left(p - c_2 - r + \theta_1\right) + \frac{N - \tilde{N}}{N}\left(p - c_2 + \theta_2\right) \tag{A.1}$$

A.1. Carbon Tax or Laissez Faire

Under a carbon tax or *laissez faire*, as discussed in the main text, firms face a horizontal demand curve with $p = c_1 + t$ (for *laissez faire*, $t = 0$). A producing firm will accept a license offer if $R_1 \geq R_2$:

$$\frac{\left(c_1 + t - c_2 - r + \theta_1\right)^2}{2N} - F \geq \frac{\left(c_1 + t - c_2 + \theta_2\right)^2}{2N} \tag{A.2}$$

The optimal two-part tariff therefore solves:

$$\max_{0 < \tilde{N} \leq N\,,\, r \geq 0\,,\, F \geq 0} \left\{ \tilde{N}\left(F + r\frac{c_1 + t - c_2 - r + \theta_1}{N} \right) \right\} \tag{A.3}$$

conditional on the participation constraint in equation (A.2).

Because the number of licenses has no impact on the output price under a carbon tax, it is optimal for the innovator to fully license, so that $\tilde{N} = N$. At this point, the usual two-part tariff solution applies. The optimal policy sets $r^* = 0$ to maximize each firm's producer surplus and then appropriates as much of it as feasible:

$$F^* = \frac{\left(c_1 + t - c_2 + \theta_1\right)^2}{2N} - \frac{\left(c_1 + t - c_2 + \theta_2\right)^2}{2N} \tag{A.4}$$

To make this solution comparable to the paper's analysis, set $c_2 = c_1 + t$ (consistent with Condition 3 in the paper). The total licensing profit with a two-part tariff, $\pi_t^{II} = NF^*$, is then:

$$\pi_t^{II} = \frac{\theta_1^2 - \theta_2^2}{2} = \frac{\left(\theta_1 + \theta_2\right)\left(\theta_1 - \theta_2\right)}{2} \tag{A.5}$$

For comparison, the licensing profit with the royalty-only license in the paper [equation (19)] is:

$$\pi_t = \begin{cases} \left(\theta_1/2\right)^2 & \text{if } \theta_2 \le \theta_1/2 \\ \left(\theta_1 - \theta_2\right)\theta_2 & \text{if } \theta_2 > \theta_1/2 \end{cases} \tag{A.6}$$

It can be verified the innovator makes strictly more from a two-part tariff for any realization of θ_1 and θ_2. Meanwhile, the renewable energy firms are no worse off, because they capture the producer surplus they would have achieved freely using the second-best technology θ_2 – the same as in the main text of paper. However, aggregate welfare is improved by the efficient use of renewable energy (because marginal cost with no royalty is equated to price).

The expected two-part tariff licensing profit $\pi_t^{II}\left(\omega, n\right)$ of each innovator facing technological opportunity ω when there are n innovators engaged in R&D can then be expressed as:

$$\pi_t^{II}\left(\omega, n\right) = \int_0^{\omega} \left\{ \int_0^{\theta_1} \left(\frac{n-1}{\theta_2}\right)\left(\frac{\theta_2}{\theta_1}\right)^{n-1} \frac{\theta_1^2 - \theta_2^2}{2} d\theta_2 \right\} \left(\frac{\theta_1}{\omega}\right)^{n-1} \frac{1}{\omega} d\theta_1 \tag{A.7}$$

Upon integration, we obtain:

$$\pi_t^{II}\left(\omega, n\right) = \frac{\omega^2}{\left(n+1\right)\left(n+2\right)} \tag{A.8}$$

For comparison, the corresponding expression with per-unit royalties in the main text of the paper [equation (21)] is:

$$\pi_t(\omega, n) = \frac{n - \left(1 - (1/2)^n\right)}{n(n+1)(n+2)}\omega^2 \tag{A.9}$$

and so $\pi_t^{II}(\omega, n) > \pi_t(\omega, n)$. Combined with the free-entry (zero profit) condition, we can assert there will be at least n entrants, under a carbon tax with a two-part tariff, whenever $\omega > \hat{\omega}_t^{II}(n)$, where $\hat{\omega}_t^{II}(n)$ is:

$$\hat{\omega}_t^{II}(n) = \sqrt{(n+1)(n+2)k} \tag{A.10}$$

A2. Mandates

To simplify notation for this section, define:

$$\Delta \equiv \theta_1 - \theta_2$$

$$c \equiv c_2 - \theta_2$$

$$\alpha \equiv \tilde{N}/N$$

Using this notation, the producing firms' returns from using technologies θ_2 and θ_1 , therefore, are:

$$R_2 = \frac{(p-c)^2}{2N} \quad \text{and} \quad R_1 = \frac{(p-c+\Delta-r)^2}{2N} - F$$

Under a binding mandate, the participation constraint still requires $R_1 \geq R_2$, and therefore $r \leq \Delta$. Furthermore, for any given royalty rate $r \in [0, \Delta]$, the maximum fee that can be extracted is

$$F = \frac{(\Delta - r)^2 + 2(p-c)(\Delta - r)}{2N}.$$

Now, with $\tilde{N} \leq N$ licenses the innovators licensing profit is $\pi = \tilde{N}(rq_1 + F)$ where q_1 is the output of a firm that licenses the technology θ_1, that is: $q_1 = (p - c + \Delta - r)/N$. Hence, collecting terms:

$$\pi = \frac{\tilde{N}}{N}\left\{ r(\Delta - r) + \frac{(\Delta - r)^2}{2} + (p-c)\Delta \right\} \tag{A.11}$$

As for the output price, under a mandate this price depends on how many firms license the technology. Inverting the aggregate supply curve of the renewable fuel industry in (A.1) yields the industry marginal cost curve which, when evaluated at the binding mandate $\bar{Q}$, yields the "supply" price faced by producers in this industry for any feasible $(r, \tilde{N})$. With the current notation, this supply price under a binding mandate is:

$$p = c - \frac{\tilde{N}}{N}\left(\Delta - r\right) + \bar{Q} \tag{A.12}$$

Using (A.12) in (A.11), and recalling that the fraction of firms licensing the new technology has been defined as $\alpha \equiv \tilde{N}/N$, the licensing profit with the two-part tariff can be expressed as:

$$\pi = \alpha\Delta\bar{Q} + \alpha\frac{(\Delta^2 - r^2)}{2} - \alpha^2\Delta(\Delta - r) \tag{A.13}$$

The innovator's problem is to maximize this licensing revenue over $\alpha \in [0,1]$ and $r \in [0,\Delta]$. The Kuhn-Tucker conditions are

$$\frac{\partial\pi}{\partial\alpha} = 0 \text{ if } \alpha^* \in (0,1) \quad ; \quad \frac{\partial\pi}{\partial\alpha} \le 0 \text{ if } \alpha^* = 0 \quad ; \quad \frac{\partial\pi}{\partial\alpha} \ge 0 \text{ if } \alpha^* = 1$$

$$\frac{\partial\pi}{\partial r} = 0 \text{ if } r^* \in (0,\Delta) \quad ; \quad \frac{\partial\pi}{\partial r} \le 0 \text{ if } r^* = 0 \quad ; \quad \frac{\partial\pi}{\partial r} \ge 0 \text{ if } r^* = \Delta$$

where

$$\frac{\partial\pi}{\partial\alpha} = \Delta\bar{Q} + \frac{(\Delta^2 - r^2)}{2} - 2\alpha\Delta(\Delta - r)$$

$$\frac{\partial\pi}{\partial r} = -\alpha r + \alpha^2\Delta$$

Quite clearly, $\alpha^* = 0$ cannot be a solution. If $r^* = \Delta$ then $\partial\pi / \partial\alpha > 0$ and it must be that $\alpha^* = 1$, and if $\alpha^* = 1$ then $\partial\pi / \partial r \ge 0$ requires that $r^* = \Delta$. Hence, the corner solution $\left(\alpha^* = 1, r^* = \Delta\right)$ does satisfy the K-T conditions, i.e., it identifies a local maximum. The other possible solution is for both r^* and α^* to be interior. In such a case, then from $\partial\pi / \partial r = 0$ we have $r^* = \alpha\Delta$. From (A.13), licensing profit, conditional on $r^* = \alpha\Delta$, can be written as:

$$\pi = \alpha\Delta\bar{Q} + \frac{\alpha\Delta^2(1-\alpha)^2}{2} \tag{A.14}$$

The FOC for an interior solution then requires

$$\frac{\partial\pi}{\partial\alpha} = \Delta\bar{Q} + \frac{\Delta^2(1-\alpha)^2}{2} - \alpha\Delta^2(1-\alpha) = 0 \tag{A.15}$$

with the SOSC

$$\frac{\partial^2\pi}{\partial\alpha^2} = -\Delta^2\left(2-3\alpha\right) < 0$$

Using the quadratic formula, from the FOC we find

$$\alpha = \frac{4\Delta \pm \sqrt{16\Delta^2 - 12\Delta(2\bar{Q}+\Delta)}}{6\Delta} \quad = \quad \frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{1}{3}\left(\frac{2\bar{Q}}{\Delta}+1\right)}$$

Note that for there to exist an interior local maximum solution we need

$$\frac{4}{9} - \frac{1}{3}\left(\frac{2\bar{Q}}{\Delta\theta}+1\right) > 0 \quad \rightarrow \quad \Delta\theta > 6\bar{Q}$$

Furthermore, because the SOSC for such a putative optimum requires $\alpha < 2/3$, we take the smaller root, and thus the interior solution of interest is:

$$\tilde{\alpha} = \frac{2}{3} - \frac{1}{3}\sqrt{4 - 3\left(\frac{2\bar{Q}}{\Delta}+1\right)} \tag{A.16}$$

Note that, from (A.15), $\left.\frac{\partial\pi}{\partial\alpha}\right|_{\alpha=1} > 0$, which again shows that the corner solution $\alpha = 1$ is always a local maximum. Hence, the question is whether the interior local maximum solution in (A.16) is a global maximum. For this, we need that $\pi|_{\tilde{\alpha}} > \pi|_{\alpha=1}$, that is:

$$\left(\frac{2}{3} - \frac{1}{3}\sqrt{4-3\left(\frac{2\bar{Q}}{\Delta}+1\right)}\right)\Delta\bar{Q} + \frac{\left(\frac{2}{3} - \frac{1}{3}\sqrt{4-3\left(\frac{2\bar{Q}}{\Delta}+1\right)}\right)\Delta^2\left(\frac{1}{3} + \frac{1}{3}\sqrt{4-3\left(\frac{2\bar{Q}}{\Delta}+1\right)}\right)^2}{2} > \Delta\bar{Q}$$

or, equivalently:

$$\left(2-\sqrt{4-3\left(\frac{2\bar{Q}}{\Delta}+1\right)}\right)\left(1+\sqrt{4-3\left(\frac{2\bar{Q}}{\Delta}+1\right)}\right)>9\left(\frac{2\bar{Q}}{\Delta}\right)$$

Some further algebra can reduce this condition for an interior global maximum to

$$\Delta > 8\bar{Q}$$

Hence, for $\Delta \equiv (\theta_1 - \theta_2) > 8\bar{Q}$, profit is maximized by a partial licensing solution $\tilde{N} < N$. Conversely, if $(\theta_1 - \theta_2) \le 8\bar{Q}$ then licensing profit under a mandate is maximized by the full licensing solution considered in the paper, that is $r^* = \theta_1 - \theta_2$, $\tilde{N} = N$ and $F^* = 0$.

Note that, in the logic of the model, $\theta_1 - \theta_2 \le \bar{\varpi}$. Also, Condition 2 in the paper requires $\bar{\varpi} - (c_2 - c_1) \le \bar{Q}$. Hence, the condition $\theta_1 - \theta_2 \le \bar{Q} + (c_2 - c_1)$ is already satisfied. Thus, the full licensing solution will obtain if $\bar{Q} + (c_2 - c_1) \le 8\bar{Q}$, or $(c_2 - c_1)/7 \le \bar{Q}$. Hence, a sufficient condition to rule out any scope for incomplete licensing and/or a two-part tariff, under a mandate, would be to replace Condition 2 in the text with the following:

Condition 2A. The mandate is large enough to always bind and require full licensing under a two-part tariff, i.e., $\bar{Q} \ge \max\left[\bar{\varpi} - (c_2 - c_1), (c_2 - c_1)/7\right]$.

A.3. Comparing Carbon Taxes and Mandates under a Two-Part Tariff

In conclusion, the qualitative results obtained with the per-unit royalty model of the paper carry through under the more general two-part tariff licensing scheme. Use of a two-part tariff improves the licensing prospect of an innovator under a carbon tax, but it is far less likely to do so under a binding mandate. Indeed, Condition 2A would rule out any scope for incomplete licensing and/or two part tariff under a mandate. In such a case, it is more likely there will be at least n innovators under a carbon tax than under a mandate whenever $\hat{\omega}_m(n) \ge \hat{\omega}_t^{II}(n)$. By using equations (A.10) and the definition of $\hat{\omega}_m(n)$ given in the paper, and simplifying, this condition reduces to:

$$\frac{k}{\bar{Q}^2} > \frac{n+2}{n^2(n+1)} \tag{A.17}$$

which can be compared with the condition in equation (28) in the paper. For any given policy, the left hand side of (A.17) is fixed, while the right hand side is decreasing in n. This implies there is a threshold $\hat{n}$ such that $\hat{\omega}_m(n) \geq \hat{\omega}_t(n)$ whenever $n > \hat{n}$, where $\hat{n}$ is now defined by:

$$\frac{\hat{n}+1}{(\hat{n}-1)^2\hat{n}} \geq \frac{k}{\hat{Q}^2} \geq \frac{\hat{n}+2}{\hat{n}^2(\hat{n}+1)} \tag{A.18}$$

Because $\hat{\omega}_m(n) \geq \hat{\omega}_t(n)$ for all $n \geq \hat{n}$, and given that $\hat{\omega}_m(n)$ and $\hat{\omega}_t(n)$ are monotonically increasing in n, we conclude that RESULT 3 in the paper still applies, except that $\hat{\omega}_t^{II}(n)$ replaces $\hat{\omega}_t(\hat{n})$. The rest of the results also go through with little alteration, except that the mandate level such that $\hat{\omega}_t(1) = \hat{\omega}_m(1)$ (the single innovator case) would now be $\bar{Q} = \sqrt{2k/3}$ (rather than $\bar{Q} = \sqrt{k/3}$ for the case analyzed in the text).

APPENDIX B. MULTIPLE-PROJECT INNOVATORS

The model of the paper has developed an "idea" framework where each active firm pursues a distinct R&D project upon incurring the fixed cost k. As noted by a reviewer, one might want to consider the possibility that the investment level of the firm is itself endogenous. Following Spulber (2013), this can be accomplished in our framework by allowing firms to undertake multiple R&D projects. In this Appendix we develop this approach for the case of a monopolist innovator. We find that the core results reported in the paper also obtain in this alternative model where a monopolist has the ability to select n distinct research programs, each of which costs k to implement and yields an independent draw from $F(\theta \mid \omega)$.

B.1. Mandates

Under a mandate, the monopolist's licensing revenues are $\theta_1 \bar{Q}$ where θ_1 is the maximum of n draws from $F(\theta \mid \omega)$. Given the order statistics discussed in the paper, expected licensing revenues from n projects are:

$$\pi_m^I(\omega, n) = \int_0^\omega \theta \bar{Q} n \left(\frac{\theta}{\omega}\right)^{n-1} \frac{1}{\omega} d\theta \tag{B.19}$$

Performing the integration yields:

$$\pi_m^I(\omega, n) = \frac{n\bar{Q}}{n+1}\omega \tag{B.20}$$

The expected increase in profit from the n^{th} project is:

$$\pi_m^I(\omega, n) - \pi_m^I(\omega, n-1) = \frac{\bar{Q}}{n(n+1)}\omega \tag{B.21}$$

Note this is the same as the expected profit under a competitive innovation contest, as in equation (25) in the paper. The monopolist will increase the number of R&D projects until:

$$\pi_m^I(\omega, n) - \pi_m^I(\omega, n-1) \geq k \geq \pi_m^I(\omega, n+1) - \pi_m^I(\omega, n) \tag{B.22}$$

Under a binding mandate, a monopolist will choose the same number of projects as under the competitive innovation case analyzed in the main text. Profit maximization implies there are *at least* n active R&D projects iff $\omega \geq \hat{\omega}_m^I(n)$ where:

$$\hat{\omega}_m^I(n) = \frac{n(n+1)}{\bar{Q}}k \tag{B.23}$$

To compute expected welfare in this case, we modify equation (7) in the text by using equation (B.20) for the expected licensing profits $E[\pi_m^I]$:

$$E[W] = S_0^m + \Pi_0^m - X_0^m + \sum_{n=1}^{\bar{n}} \int_{\hat{\omega}_m^I(n)}^{\hat{\omega}_m^I(n+1)} \left(\frac{n\bar{Q}}{n+1}\omega - nk \right) dG(\omega) \tag{B.24}$$

where $\bar{n}$ is the solution to (B.22) under the best possible research outlook $\bar{\omega}$.

We can now show that RESULT 1 in the paper continues to hold under the current multiple-project framework. When the innovator can choose the number of R&D projects, an optimal mandate in this setting solves:

$$\frac{\partial E[W]}{\partial \bar{Q}} = \frac{\partial S_0^m}{\partial \bar{Q}} + \frac{\partial \Pi_0^m}{\partial \bar{Q}} - \frac{\partial X_0^m}{\partial \bar{Q}} + \frac{\partial E[\pi_m^I]}{\partial \bar{Q}} = 0 \tag{B.25}$$

The last term, the impact of changing the mandate on expected licensing profit, captures the role of innovation. Differentiating the last term in (B.24) yields:[1]

$$\frac{\partial E[\pi_m^I]}{\partial \bar{Q}} = \sum_{n=1}^{\bar{n}} \int_{\hat{\omega}_m^I(n)}^{\hat{\omega}_m^I(n+1)} \frac{n}{n+1} \omega dG(\omega) \tag{B.26}$$

which makes it apparent that the last term in (B.25) is positive. Using the same line of argument as for the proof of RESULT 1 in the main text, it then follows that RESULT 1 continues to hold when the monopolist can choose the number of R&D projects to initiate.

[1] The use of Leibniz rule in taking this derivative also yields additional terms because changing $\bar{Q}$ changes the limits of integration $\hat{\omega}_m^I(n)$ and $\hat{\omega}_m^I(n+1)$. But it is verified that these terms wash out, essentially as a consequence of the envelope theorem (the innovator selects the optimal number of projects for any given $\bar{Q}$), such that only the direct effect remains.

B.2. Carbon Tax

Under a carbon tax, the monopolist's licensing revenues are $(\theta_1/2)^2$ where θ_1 is the maximum of n draws from $F(\theta|\omega)$. Expected licensing revenues from n projects are:

$$\pi_t^I(\omega,n) = \int_0^{\omega} \left(\frac{\theta}{2}\right)^2 n\left(\frac{\theta}{\omega}\right)^{n-1} \frac{1}{\omega} d\theta \tag{B.27}$$

Performing the integration yields:

$$\pi_t^I(\omega,n) = \frac{n\omega^2}{4(n+2)} \tag{B.28}$$

The expected increase in profit from the n^{th} project is:

$$\pi_t^I(\omega,n) - \pi_t^I(\omega,n-1) = \frac{\omega^2}{2(n+2)(n+1)} \tag{B.29}$$

After the first project, this is less than the expected profit under competitive innovation. Unlike the case of binding mandates, therefore, with a carbon tax a monopolist will engage in less R&D than a competitive sector. The monopolist will increase the number of R&D projects until:

$$\pi_t^I(\omega,n) - \pi_t^I(\omega,n-1) \geq k \geq \pi_t^I(\omega,n+1) - \pi_t^I(\omega,n) \tag{B.30}$$

Profit maximization implies there are *at least* n active R&D projects iff $\omega \geq \hat{\omega}_t^I(n)$ where:

$$\hat{\omega}_t^I(n) = \sqrt{2k(n+2)(n+1)} \tag{B.31}$$

To compute expected welfare in this case, we modify equation (13) in the main text by using equation (B.28) for the expected licensing profits, noting that the producer surplus of renewable energy producers can be shown to be equal to $n\omega^2/(8(n+2))$:

$$E[W] = S_0^* + \sum_{n=1}^{\bar{n}} \int_{\hat{\omega}_t^I(n)}^{\hat{\omega}_t^I(n+1)} \left(\frac{n\omega^2}{4(n+2)} + \frac{n\omega^2}{8(n+2)} - nk \right) dG(\omega) \tag{B.32}$$

We can now show that a version of RESULT 2 stated in the paper continues to hold, specifically:

RESULT B2. When the mandate is such that the probability of some R&D under a mandate is equal to the probability of some R&D under a carbon tax, then expected welfare is higher with the carbon tax.

As in Result 2 in the text, the focus here is on ensuring the same probability—under either a carbon tax or a mandate—that at least one R&D project is undertaken. To prove RESULT B2, note that $\bar{Q} = \sqrt{k/3}$ continues to ensure that $\hat{\omega}_m^I(1) = \hat{\omega}_t^I(1)$. As argued with RESULT 2 in the paper, for the proof it suffices to show the component of expected welfare due to innovation is higher under a carbon tax than under a mandate, that is:

$$\sum_{n=1}^{\bar{n}_t} \int_{\hat{\omega}_t^I(n)}^{\hat{\omega}_t^I(n+1)} \left(\frac{n\omega^2}{4(n+2)} + \frac{n\omega^2}{8(n+2)} - nk \right) dG(\omega) \geq \sum_{n=1}^{\bar{n}_m} \int_{\hat{\omega}_m^I(n)}^{\hat{\omega}_m^I(n+1)} \left(\frac{n\bar{Q}}{n+1}\omega - nk \right) dG(\omega) \qquad \text{(B.33)}$$

where $\bar{n}_t$ and $\bar{n}_m$ are the optimal number of projects under the carbon tax and the mandate, respectively, for the best possible research outlook $\bar{\omega}$, and where, as noted, $\hat{\omega}_m^I(1) = \hat{\omega}_t^I(1)$. As in the paper, if the left-hand integrand is everywhere greater than the right-hand side equation (B.33) will hold. Let n_i^* $(i = t, m)$ denote the number of R&D projects initiated for a given ω. Then a sufficient condition to ensure the desired condition is that, conditional on a given ω, the profit obtained under a carbon tax exceeds the profit under a mandate (i.e., ignoring producer surplus), that is:

$$\frac{n_t^*\omega^2}{4(n_t^*+2)} - n_t^*k \geq \frac{n_m^*\bar{Q}}{n_m^*+1}\omega - n_m^*k \qquad \text{(B.34)}$$

To establish this result we proceed through the following chain of inequalities:

$$\frac{n_t^*\omega^2}{4(n_t^*+2)} - n_t^*k \quad \geq \quad \frac{n_m^*\omega^2}{4(n_m^*+2)} - n_m^*k \quad \geq \quad \frac{n_m^*\bar{Q}}{n_m^*+1}\omega - n_m^*k \qquad \text{(B.35)}$$

The first of these inequalities asserts that the expected licensing profit under a carbon tax (the left-hand-side term) is no lower when the innovator uses the number of projects n_t^* that is actually optimal under this policy, rather than relying on another number of projects such as n_m^* — an obvious implication of profit maximization. As for the second inequality in (B.35), we can show that this will hold for $\bar{Q} = \sqrt{k/3}$ which, as noted above, is the mandate level that ensure that carbon tax and mandates yield the same probability of one R&D project, i.e., $\hat{\omega}_m^I(1) = \hat{\omega}_t^I(1)$. With $\bar{Q} = \sqrt{k/3}$, the second inequality in (B.35) simplifies to:

$$\omega \geq \frac{4(n_m^* + 2)}{n_m^* + 1}\sqrt{\frac{k}{3}} \tag{B.36}$$

The right-hand side of equation (B.36) obtains a maximum of $\sqrt{12k}$ at $n_m^* = 1$. Of course, for $\omega < \sqrt{12k}$ the optimal number of R&D projects is zero under both a carbon tax or a mandate, and so either policy obtains the same (zero) profit. This establishes that the inequality chain in (B.35) holds everywhere, and consequently the desired condition in (B.34) also holds, which suffices to prove RESULT B2.

B.3. Comparing Carbon Taxes and Mandates

Some of the qualitative results obtained in the competitive innovation section of the paper have an analogous version in the current setting with a multi-project monopolist. In particular, we can verify the following version of Result 3 reported in the paper:

> **RESULT B3**. Whenever technological opportunity exceeds a certain threshold, $\omega \geq \hat{\omega}_t^I(\hat{n})$, the number of projects undertaken by the innovator is (weakly) higher under a carbon tax than under a mandate. Conversely, whenever $\omega \leq \hat{\omega}_t^I(\hat{n})$, the number of projects is (weakly) higher under a mandate policy than a carbon tax.

Recall that it is more likely there will be at least n innovators under a carbon tax than under a mandate whenever $\hat{\omega}_m^I(n) \geq \hat{\omega}_t^I(n)$. By using equations (B.23) and (B.31), and simplifying, this condition reduces to:

$$\frac{k}{2\bar{Q}^2} > \frac{n+2}{n^2(n+1)} \tag{B.37}$$

which can be compared with the condition in equation (28) in the paper. For any given policy, the left hand side of (B.37) is fixed, while the right hand side is decreasing in n. This implies there is a threshold $\hat{n}$ such that $\hat{\omega}_m^I(n) \geq \hat{\omega}_t^I(n)$ whenever $n > \hat{n}$, where $\hat{n}$ is now defined by:

$$\frac{\hat{n}+1}{(\hat{n}-1)^2\hat{n}} \geq \frac{k}{2\bar{Q}^2} \geq \frac{\hat{n}+2}{\hat{n}^2(\hat{n}+1)} \tag{B.38}$$

Because $\hat{\omega}_m^I(n) \geq \hat{\omega}_t^I(n)$ for all $n \geq \hat{n}$, and given that $\hat{\omega}_m^I(n)$ and $\hat{\omega}_t^I(n)$ are monotonically increasing in n, we conclude that RESULT 3B applies. Following the same argument as in our paper, we can also prove RESULT 4 and RESULT 5 obtain.

APPENDIX C. DETAILS OF THE NUMERICAL ANALYSIS

The numerical analysis was coded in Matlab (a copy of our code is archived at https://github.com/mattsclancy/Environmental_Innovation). The basic building block of our numerical strategy is the computation of the expected value and variance of many variables of interest. For any given set of primitive parameters (81 different parameter combinations were analyzed, as discussed in the text), there are three correlated random variables: ω (technological opportunity), θ_1 (the best realized technology), and θ_2 (the second-best realized technology). Of course, if there are more than 2 entrants, there are more than two realized draws of θ, but only the best and second-best have any impact on outcomes. The variables of interest (for example, welfare, clean energy quantity Q_2, number of innovators, etc.) depend on the realizations of these random variables. Let $y(\omega,\theta_1,\theta_2)$ denote a variable of interest that depends on the realized values of ω, θ_1, and θ_2. Using the orders statistics from section 4 of the paper and assuming ω follows a beta distribution on the interval $[0,\bar{\omega}]$, the expected value of $y(\omega,\theta_1,\theta_2)$ is:

$$E[y(\omega,\theta_1,\theta_2)] = \frac{1}{\tilde{B}}\int_0^{\bar{\omega}}\int_0^{\omega}\int_0^{\theta_1} y(\omega,\theta_1,\theta_2)\psi(\omega,\theta_1,\theta_2)d\theta_1 d\theta_2 d\omega$$

where

$$\psi(\omega,\theta_1,\theta_2) \equiv \left\{\frac{n(\omega)-1}{\theta_2}\left(\frac{\theta_2}{\theta_1}\right)^{n(\omega)-1}\right\}\left\{\frac{n(\omega)}{\theta_1}\left(\frac{\theta_1}{\omega}\right)^{n(\omega)}\right\}\left\{\left(\frac{\omega}{\bar{\omega}}\right)^{\alpha}\left(1-\frac{\omega}{\bar{\omega}}\right)^{\beta}\right\}$$

Here, $\tilde{B}$ is a factor associated with the beta distribution that normalizes its integral to 1, and $n(\omega)$ denotes the dependence of n (the number of entrants) on ω via the free-entry condition. The variance is computed similarly, by replacing $y(\omega,\theta_1,\theta_2)$ with $y^2(\omega,\theta_1,\theta_2)$.

To compute $E[y(\omega,\theta_1,\theta_2)]$, the integral in the above equation is discretized and approximated by the following:

$$E[y(\omega,\theta_1,\theta_2)] \approx \sum_{\{(\omega,\theta_1,\theta_2)\}} y(\omega,\theta_1,\theta_2)\Psi(\omega,\theta_1,\theta_2)$$

where

$$\Psi(\omega,\theta_1,\theta_2) \equiv \left\{\frac{1}{\Theta_2}\frac{n(\omega)-1}{\theta_2}\left(\frac{\theta_2}{\theta_1}\right)^{n(\omega)-1}\right\}\left\{\frac{1}{\Theta_1}\frac{n(\omega)}{\theta_1}\left(\frac{\theta_1}{\omega}\right)^{n(\omega)}\right\}\left\{\frac{1}{B}\left(\frac{\omega}{\bar{\omega}}\right)^{\alpha}\left(1-\frac{\omega}{\bar{\omega}}\right)^{\beta}\right\}$$

The function $y(\omega,\theta_1,\theta_2)$ is evaluated at a set of grid points $(\omega,\theta_1,\theta_2)$ each of which is weighted by a discretized probability (the terms B, Θ_1, and Θ_2 insure each discrete probability distribution adds up to one). The set of ω, θ_1, and θ_2 to be evaluated form a grid of evaluation points, determined in the following way.

First, divide the interval $[0,\bar{\omega}]$ into 100 equal segments. The midpoint of each segment forms a set of ω's. Next, for the Nth ω, if $n(\omega)>0$, divide the interval $[0,\omega]$ into N equal segments. Thus, the first ω "divides" the interval $[0,\omega]$ into one segment, the second divides the corresponding interval into two segment, the third into three segments and so forth. The midpoint of each section forms a set of θ_1's associated with each ω. If $n(\omega)=0$, we only consider the point $(\omega,0,0)$, which is given a probability weight

$$\frac{1}{B}\left(\frac{\omega}{\bar{\omega}}\right)^{\alpha}\left(1-\frac{\omega}{\bar{\omega}}\right)^{\beta}$$

(implicitly, we assume when $n(\omega)=0$, $\theta_1=\theta_2=0$ with certainty). If $n(\omega)>1$, then for the Nth θ_1, divide the interval $[0,\theta_1]$ into N equal segments. The midpoint of each segment form a set of θ_2's associated with the (ω,θ_1) pair. If $n(\omega)=1$, we only evaluate the point $(\omega,\theta_1,0)$, which is given a probability weight

$$\left\{\frac{1}{\Theta_1}\frac{n(\omega)}{\theta_1}\left(\frac{\theta_1}{\omega}\right)^{n(\omega)}\right\}\left\{\frac{1}{B}\left(\frac{\omega}{\bar{\omega}}\right)^{\alpha}\left(1-\frac{\omega}{\bar{\omega}}\right)^{\beta}\right\}$$

(implicitly, we assume when $n(\omega)=1$, $\theta_2=0$ with certainty).

A critical variable of interest is welfare, given that this is the target of optimal policies. In our partial equilibrium setting, welfare is the sum of consumer surplus, licensing revenues, clean producer profits, and government revenues (when there is a tax), less damages from the externality and the costs of R&D. Taking these terms in turn, to compute consumer surplus $S(\omega,\theta_1,\theta_2)$, we note that the assumed semi-log demand function $\ln Q = a - bP$ can be derived from the quasilinear utility function $U = m + u(Q)$, where m is the numeraire and $u(Q)$ takes the form

$$u(Q) = \left(Q/b\right)\left(1 + a - \ln Q\right).$$

Because the inverse demand function with the semi-log parameterization is $P(Q) = \left(a - \ln Q\right)/b$, then the net consumer surplus reduces to $u(Q) - PQ = Q/b$. Hence, the consumer surplus of interest is computed as

$$S\left(\omega, \theta_1, \theta_2\right) = Q\left(\omega, \theta_1, \theta_2\right)/b$$

where $Q\left(\omega, \theta_1, \theta_2\right)$ is the total consumption of energy (dirty and clean). If there is a binding mandate, this quantity is determined by equation (11) in the paper, after substituting in a functional form for $P\left(Q\right)$:

$$\frac{a - \ln Q\left(\omega, \theta_1, \theta_2\right)}{b} = c_1 \frac{Q\left(\omega, \theta_1, \theta_2\right) - \bar{Q}}{Q\left(\omega, \theta_1, \theta_2\right)} + \left(c_2 - \theta_1 + r\left(\omega, \theta_1, \theta_2\right) + \bar{Q}\right) \frac{\bar{Q}}{Q\left(\omega, \theta_1, \theta_2\right)}$$

Given the royalty rate $r\left(\omega, \theta_1, \theta_2\right)$, the above is solved numerically. When $Q_2 > \bar{Q}$, as happens when taxes or subsidies are used or when innovators choose or are forced to exceed the mandate, we set the costs of clean energy production, with the royalty rate included, equal to residual demand (given by equation 4 in the paper).

To find the royalty rate $r\left(\omega, \theta_1, \theta_2\right)$ we solve for the profit maximizing royalty that satisfies the constraints $Q_2 \geq \bar{Q}$ and $r \leq \theta_1 - \theta_2$. For many values of ω, θ_1, and θ_2 there are non-convexities (for example, the winning innovator might choose a very different royalty depending on whether he decides to exceed the mandate or not), and we always select the global maximum.

Given the choice of royalty and Q_2, total energy demanded Q can be inferred and the remaining quantities making up welfare are easily computed. Licensing revenues are rQ_2, clean producer profits are $Q_2^2 / 2$, dirty energy supplied is $Q_1 = Q - Q_2$, government revenues equal tQ_1, and externality damages are xQ_1.

The cost of R&D is given by $n(\omega)k$. To compute $n(\omega)$ we solve for the largest value of n such that the following condition is satisfied:

$$\int_0^{\omega}\int_0^{\theta_1} r(\omega,\theta_1,\theta_2)Q_2(\omega,\theta_1,\theta_2)\left\{\frac{n-1}{\theta_2}\left(\frac{\theta_2}{\theta_1}\right)^{n-1}\right\}\left\{\left(\frac{\theta_1}{\omega}\right)^{n-1}\right\}\frac{1}{\omega}d\theta_2 d\theta_1 > k$$

As explained earlier, we use a discrete approximation for the integral on the left-hand side of the above equation evaluated at a grid of points and . For any , the set of evaluation points is drawn with the same methodology as above.

Finally, to determine the optimal policy, we use a golden section search algorithm. For the initial bounds of a carbon tax, we look at the interval , for the mandate, we use as an upper bound the maximum feasible mandate (so that consumers consume 100% clean energy) given no innovation.